Digital Communication Systems ECS 452

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5.2 Binary Convolutional Codes

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5.2 Binary Convolutional Codes

Introduction to Binary Convolutional Codes

Binary Convolutional Codes

- Introduced by Elias in 1955
  - There, it is referred to as convolutional parity-check symbols codes.
  - Peter Elias received
    - Claude E. Shannon Award in 1977
    - IEEE Richard W. Hamming Medal in 2002
      - for "fundamental and pioneering contributions to information theory and its applications
- The encoder **has memory**.
  - In other words, the encoder is a **sequential circuit** or a **finite-state machine**.
    - Easily implemented by shift register(s).
    - The **state** of the encoder is defined as the contents of its memory.

Binary Convolutional Codes

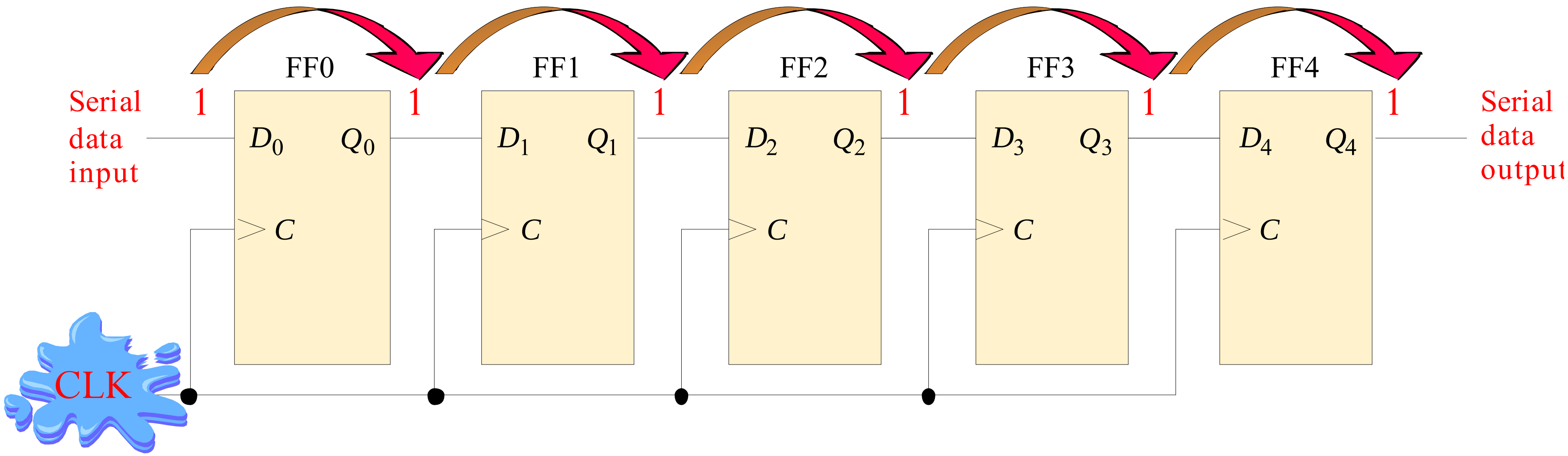
- The encoding is done on a **continuous** running basis rather than by blocks of k data digits.
  - So, we use the terms **bit streams** or **sequences** for the input and output of the encoder.
  - In theory, these sequences have infinite duration.
  - In practice, the state of the convolutional code is periodically forced to a known state and therefore code sequences are produced in a block-wise manner.

Binary Convolutional Codes

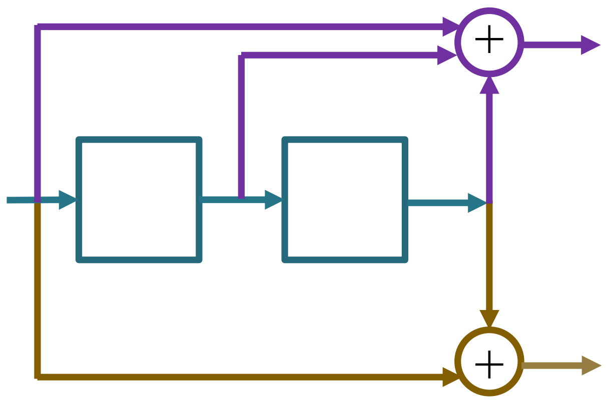
- In general, a **rate-$\frac{k}{n}$ convolutional encoder** has
  - k shift registers, one per input information bit, and
  - n output coded bits that are given by linear combinations (over the binary field, of the contents of the registers and the input information bits.
- k and n are usually small.
- For simplicity of exposition, and for practical purposes, only **rate-$\frac{1}{n}$** binary convolutional codes are considered here.
  - $k = 1$.
  - These are the most widely used binary codes.

(Serial-in/Serial-out) Shift Register

- Accept data serially: one bit at a time on a single line.
- Each clock pulse will move an input bit to the next FF. For example, a 1 is shown as it moves across.
- Example: five-bit serial-in serial-out register.



Example 1: $n = 2, k = 1$



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5.2 Binary Convolutional Codes

Three Graphical Representations

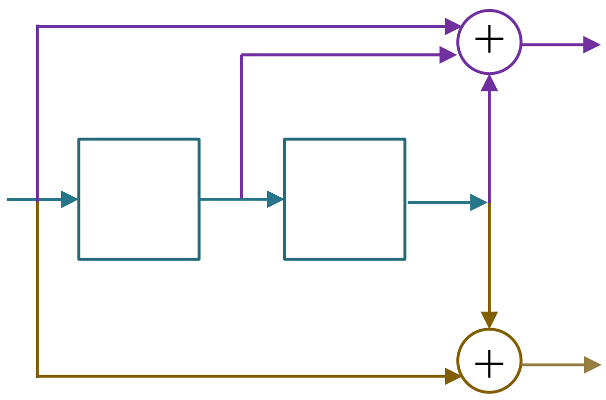
Graphical Representations

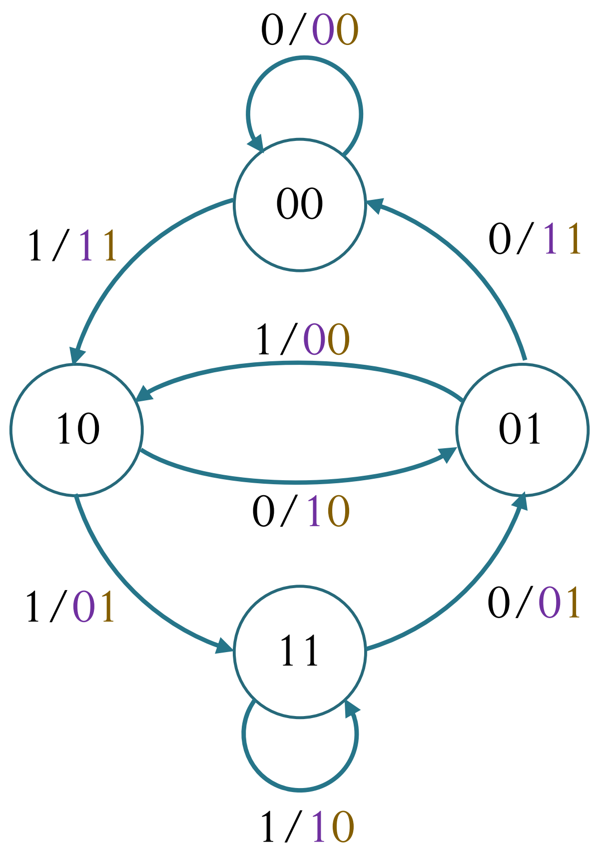
- Three different but related graphical representations have been devised for the study of convolutional encoding:

1. the state diagram
2. the code tree
3. the trellis diagram

Ex. 1: State (Transition) Diagram

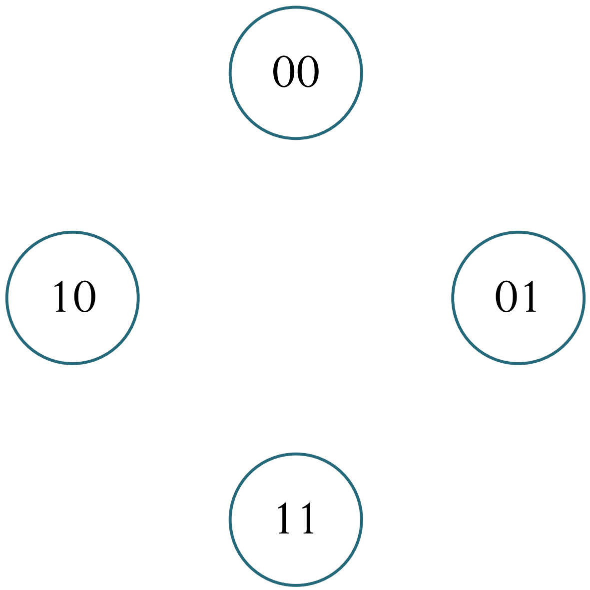
- The encoder behavior can be seen from the perspective of a finite state machine with its state (transition) diagram.

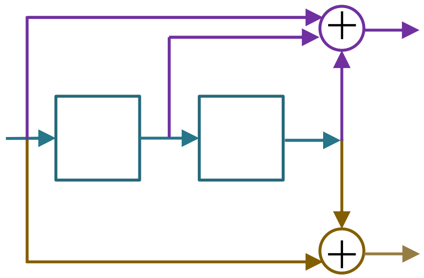




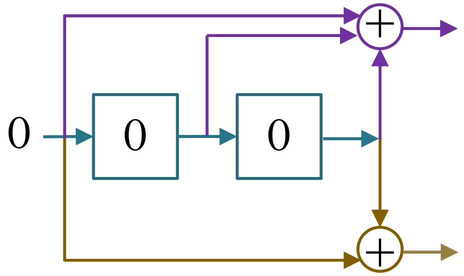
A four-state directed graph that uniquely represents the input-output relation of the encoder.

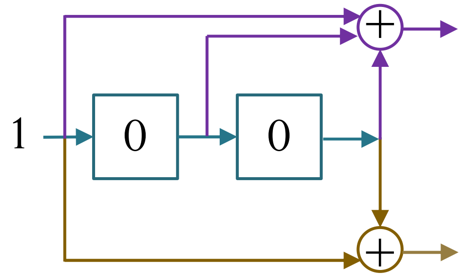
Drawing State Diagram

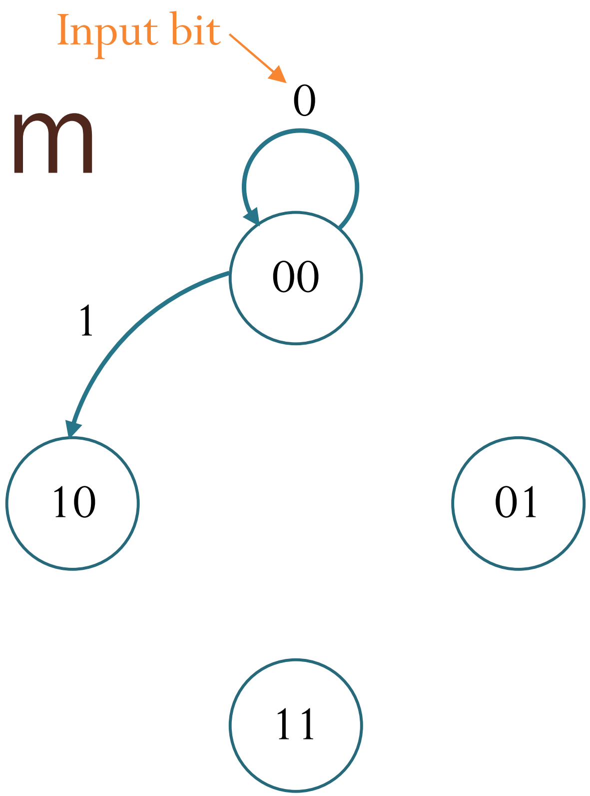


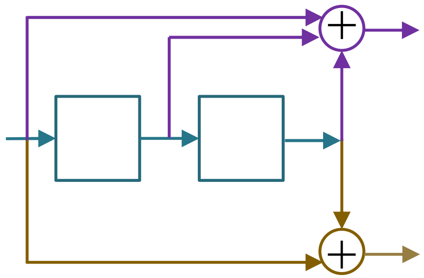


Drawing State Diagram

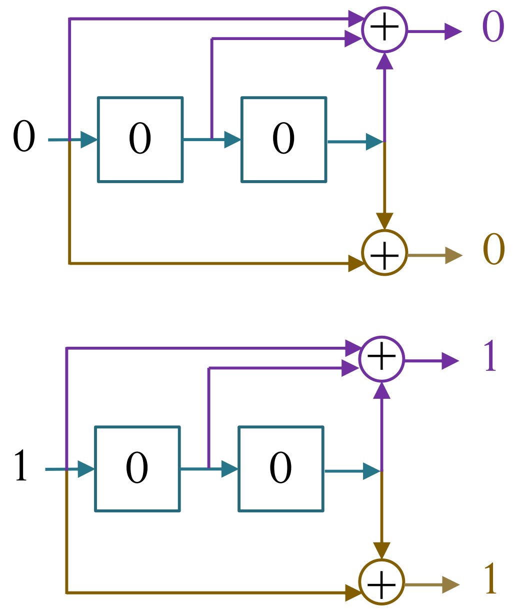


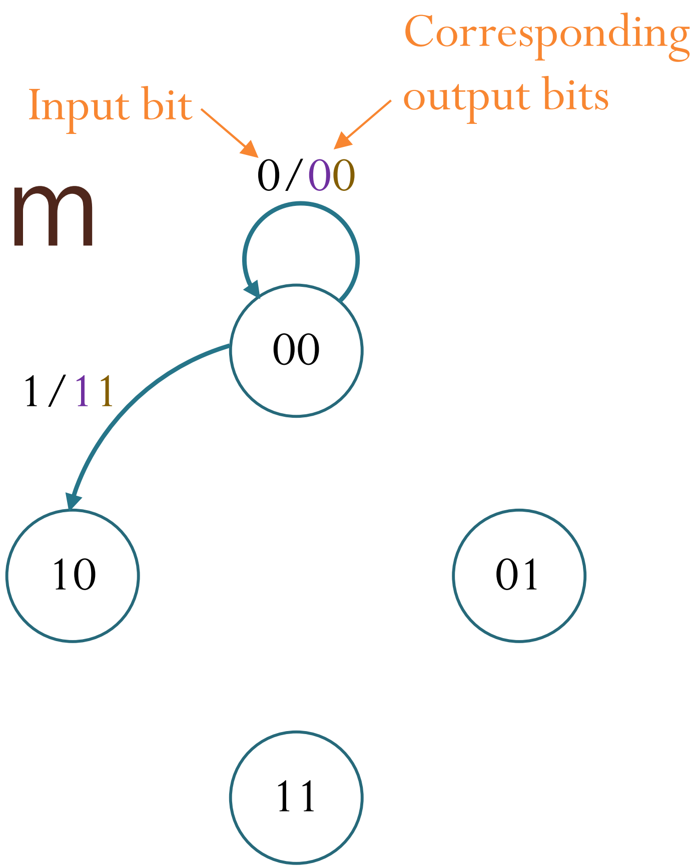


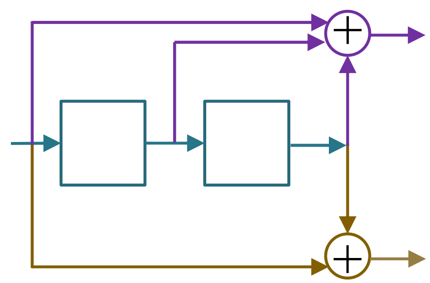




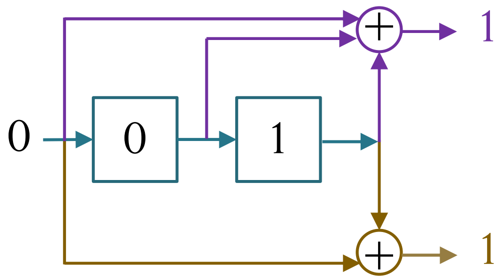
Drawing State Diagram

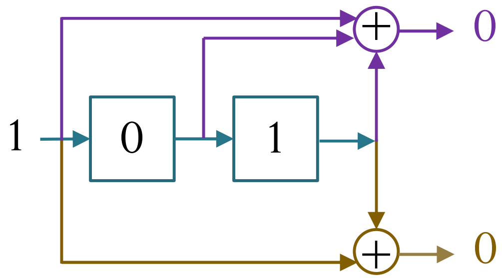


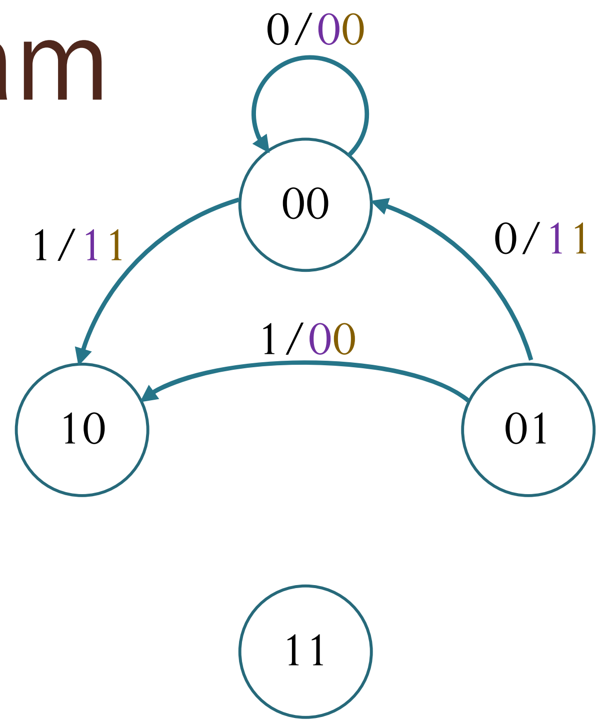


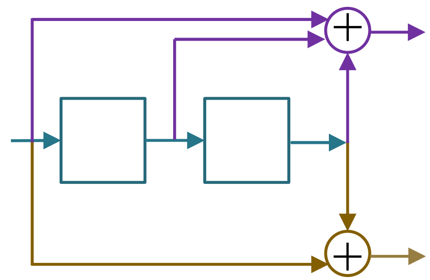


Drawing State Diagram

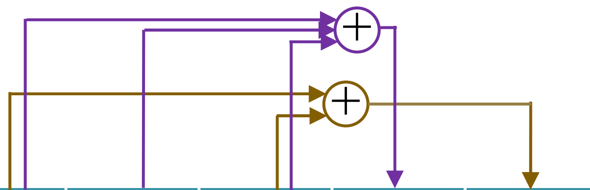




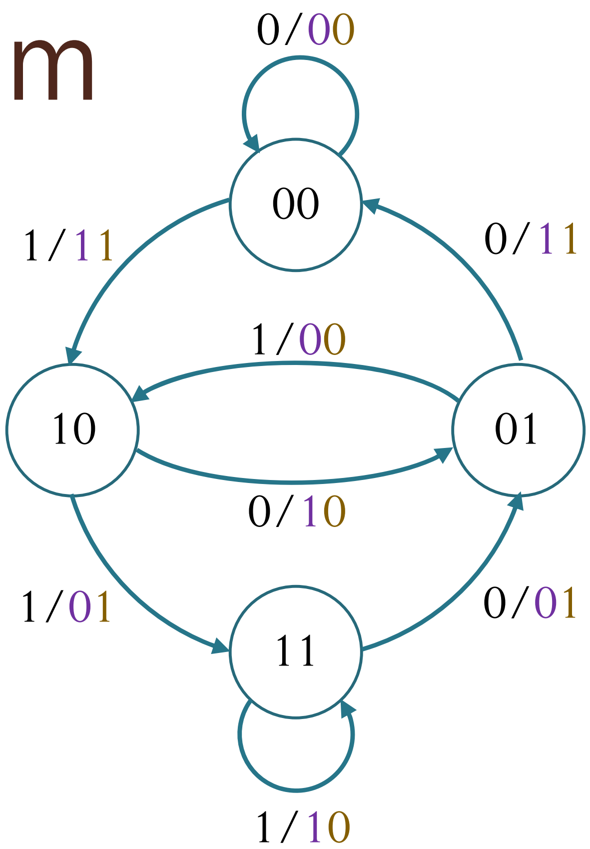


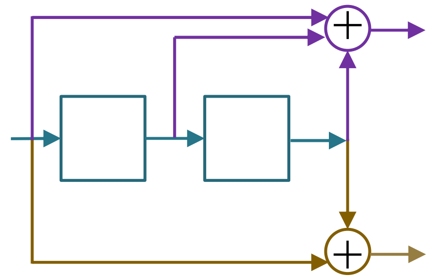


Drawing State Diagram



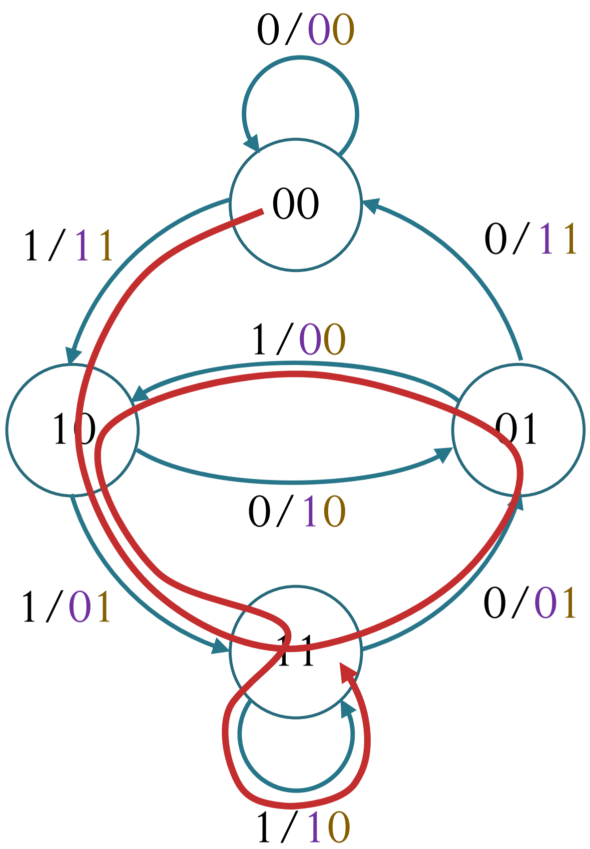
b	s_1	s_2	$x^{(1)}$	$x^{(2)}$
0	0	0	0	0
1	0	0	1	1
0	0	1	1	1
1	0	1	0	0
0	1	0	1	0
1	1	0	0	1
0	1	1	0	1
1	1	1	1	0

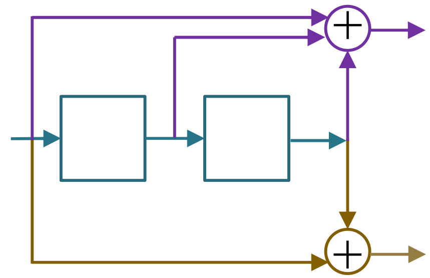




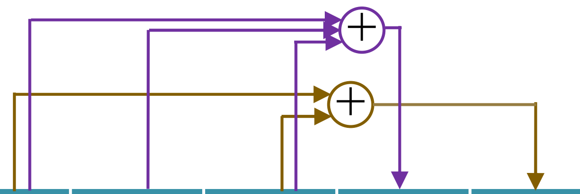
Tracing the State Diagram to Find the Outputs

Input	1	1	0	1	1	1
Output	11	01	01	00	01	10



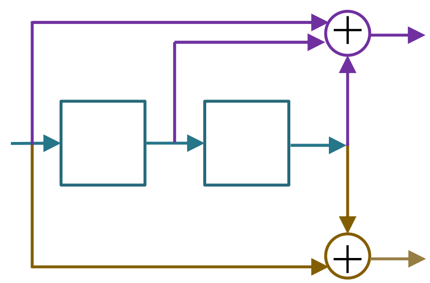


Directly Finding the Output

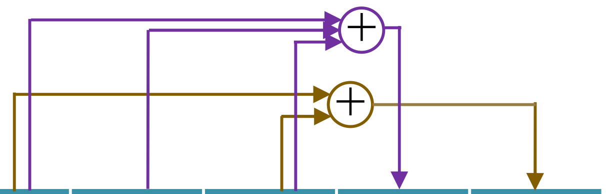


b	s_1	s_2	$x^{(1)}$	$x^{(2)}$
1	0	0		
1				
0				
1				
1				
1				

Input	1	1	0	1	1	1
Output						

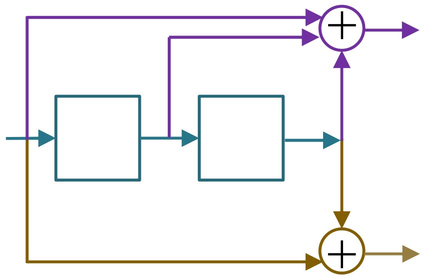


Directly Finding the Output



b	s_1	s_2	$x^{(1)}$	$x^{(2)}$
1	0	0	1	1
1	1	0	0	1
0	1	1	0	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

Input	1	1	0	1	1	1
Output	11	01	01	00	01	10



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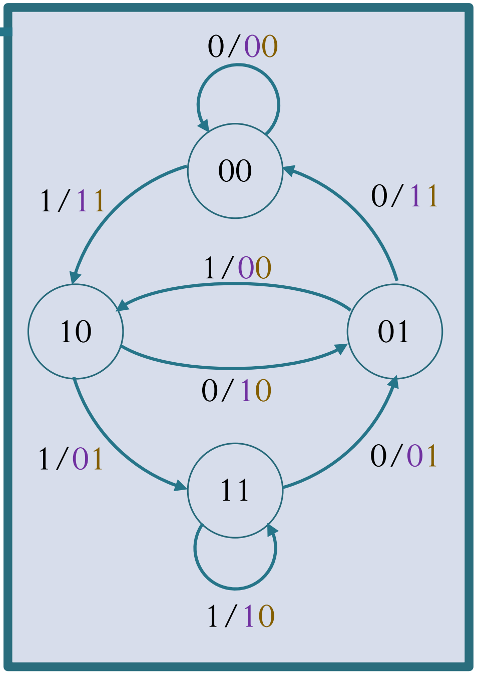
5.2 Binary Convolutional Codes

Code Tree and Trellis Diagram

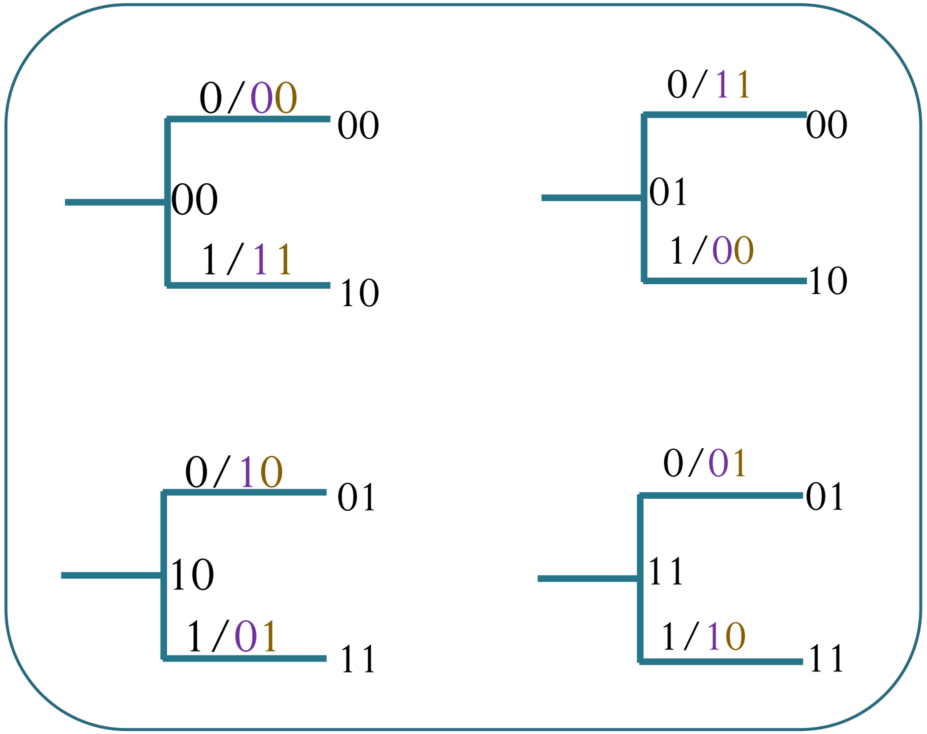
Graphical Representations

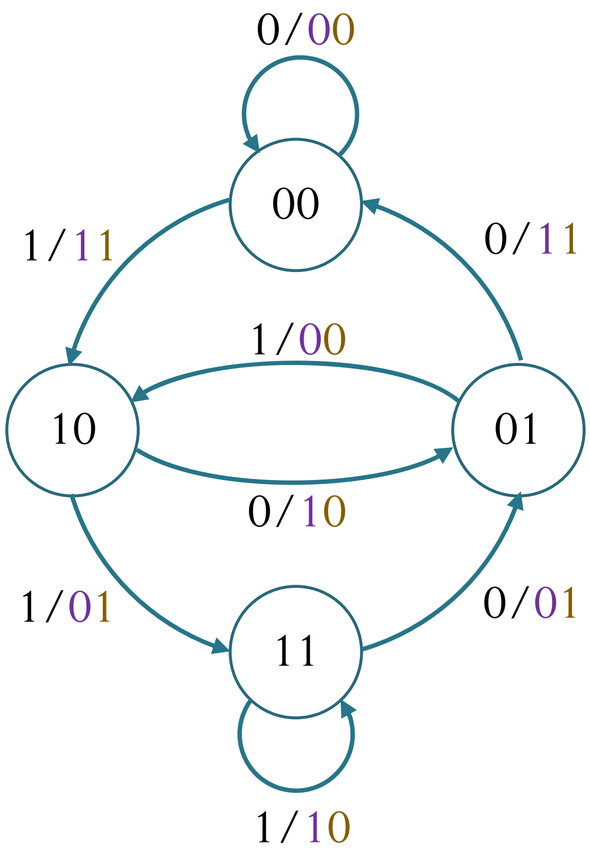
- Three different but related graphical representations have been devised for the study of convolutional encoding:

1. the state diagram
2. the code tree
3. the trellis diagram

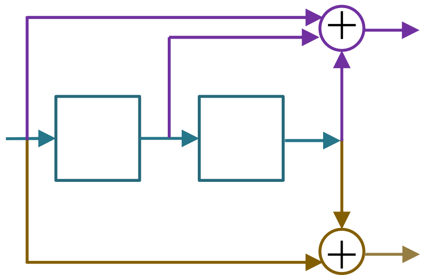


Parts for Code Tree





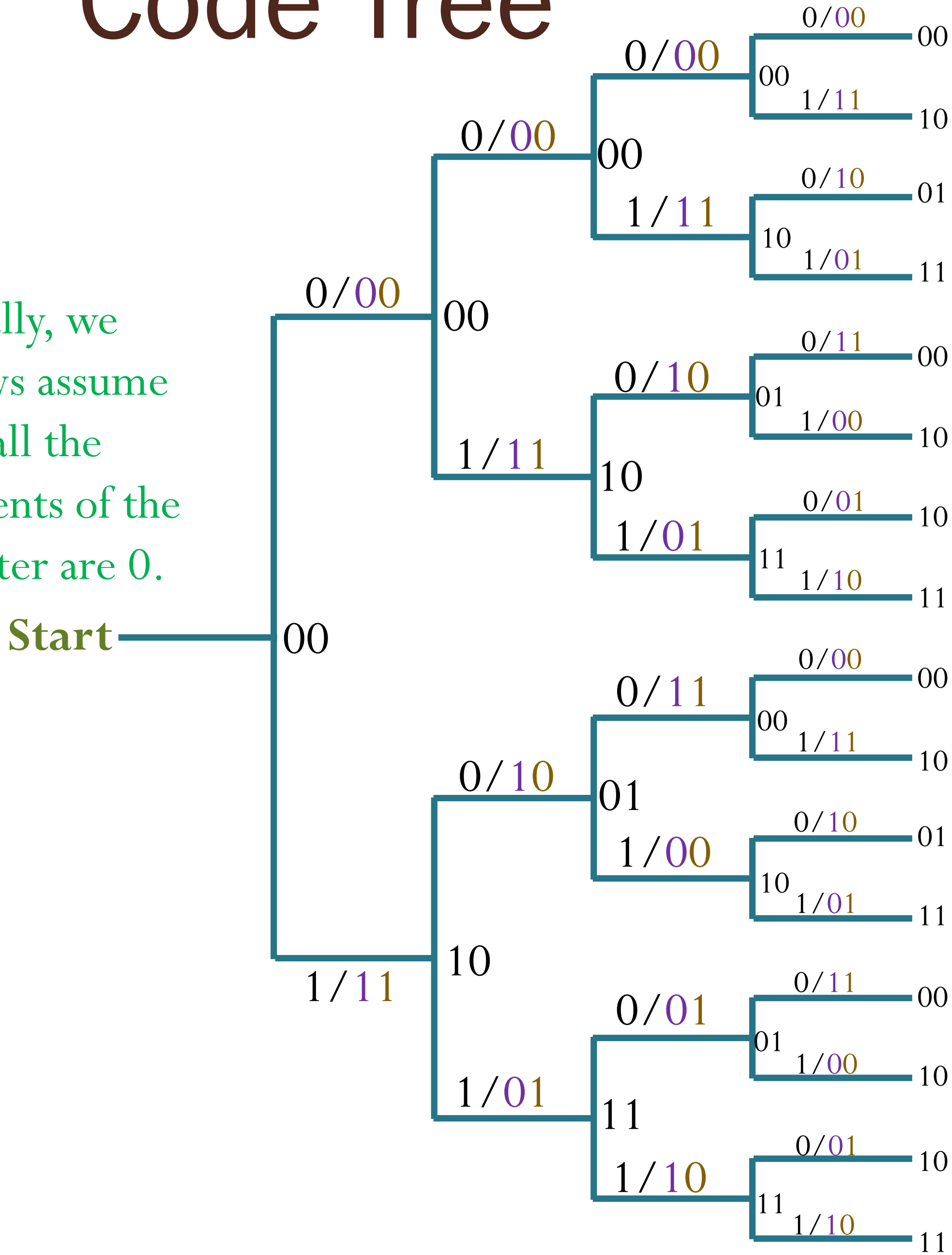
Two branches initiate from each node,
the upper one for 0 and
the lower one for 1.

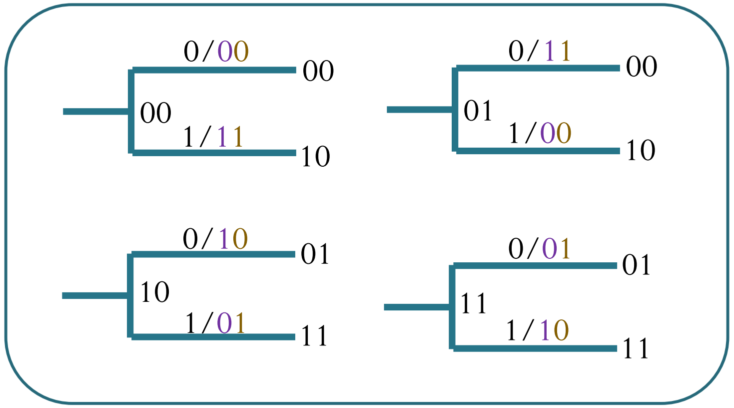


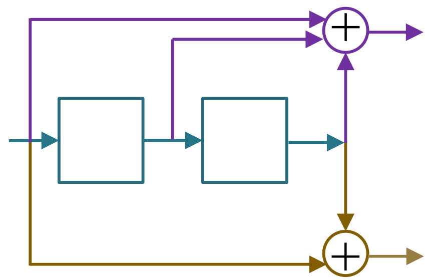
Show the coded output for any possible sequence of data digits.

Code Tree

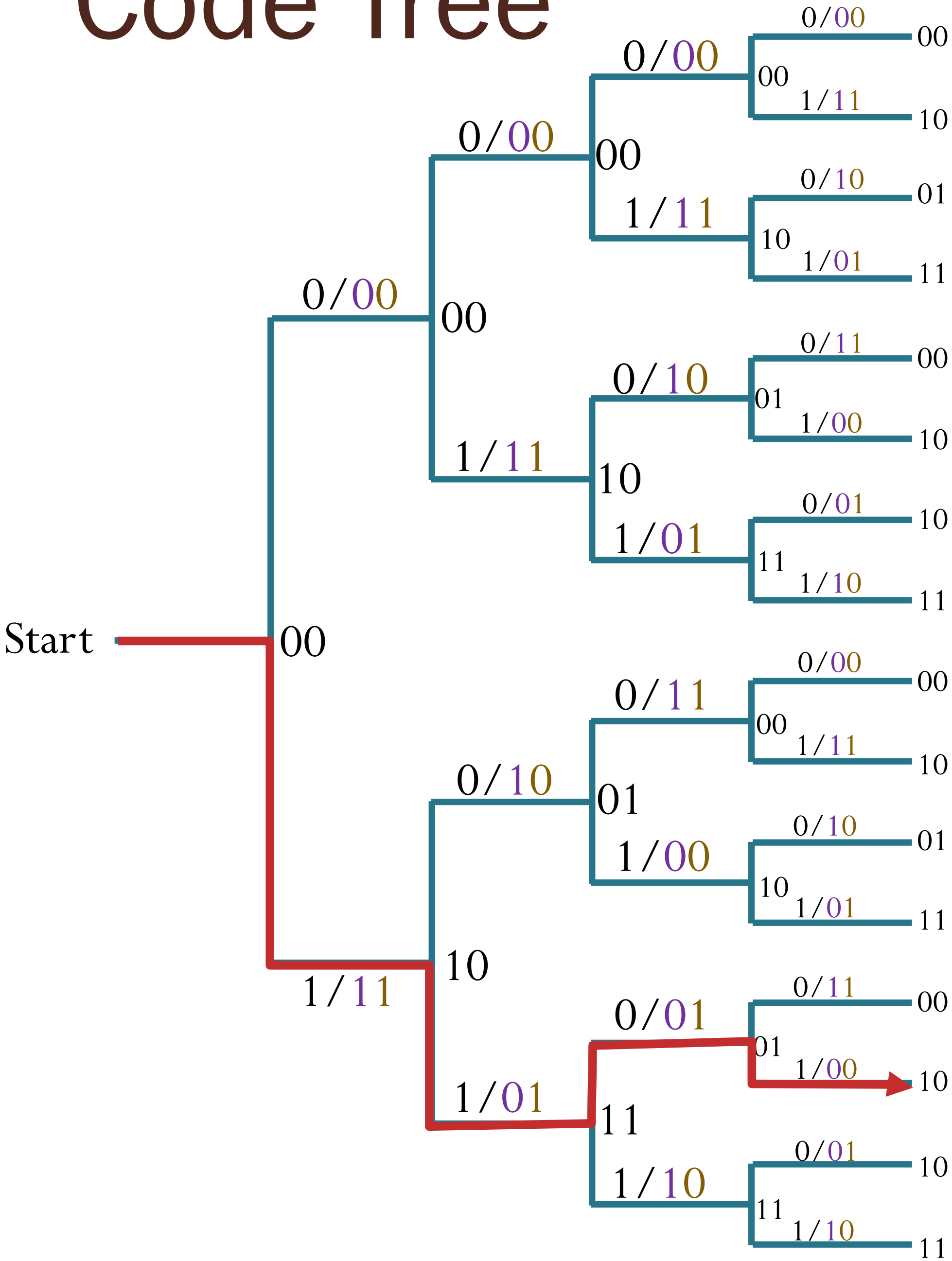
Initially, we always assume that all the contents of the register are 0.



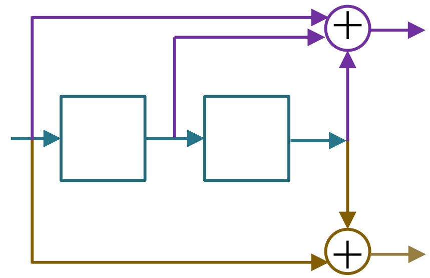




Code Tree

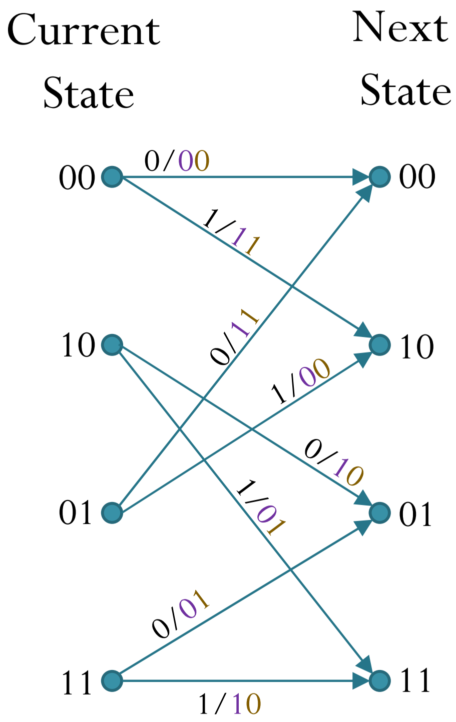


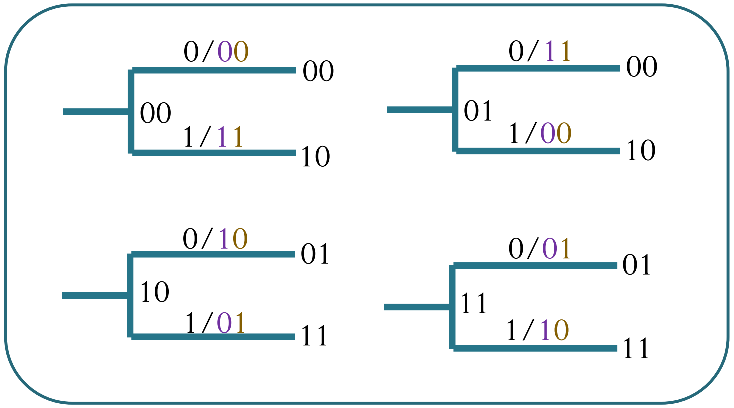
Input	1	1	0	1
Output	11	01	01	00

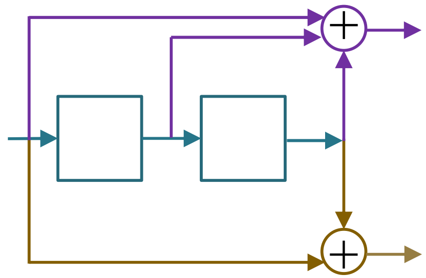


Code Trellis

[Carlson & Crilly, 2009, p. 620]

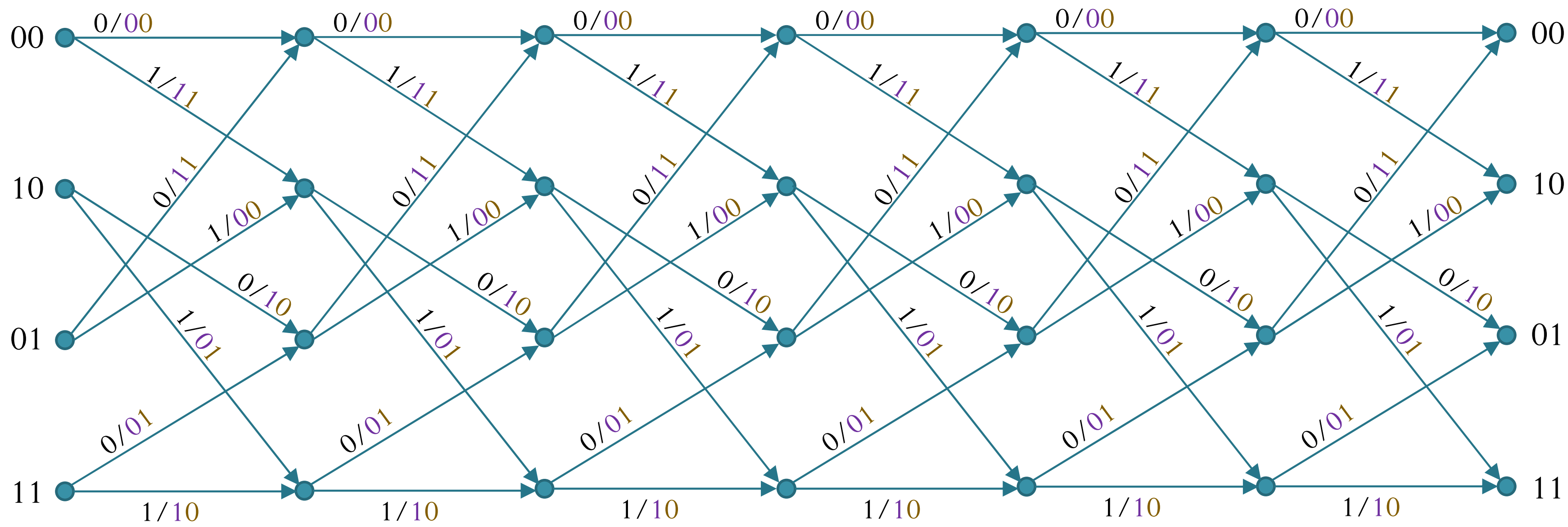


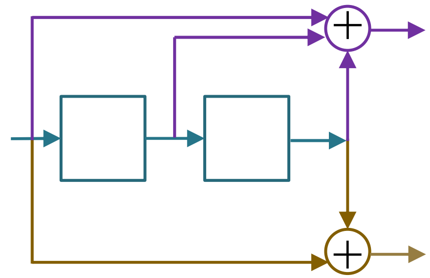




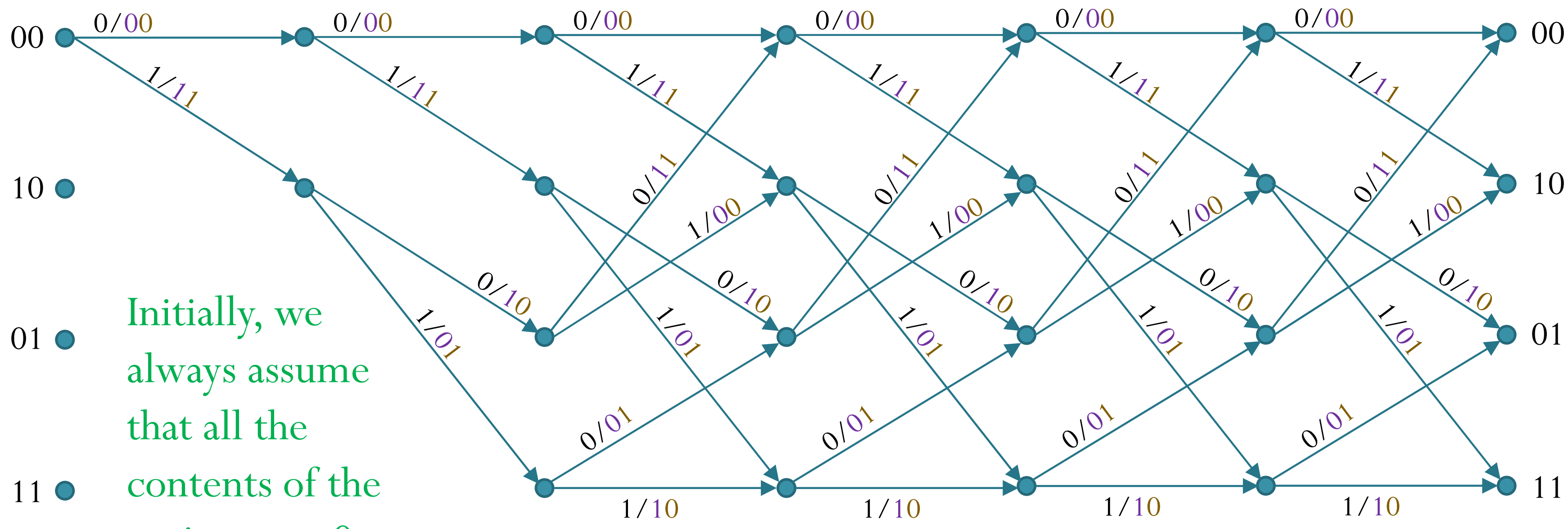
Towards the Trellis Diagram

Another useful way of representing the code tree.



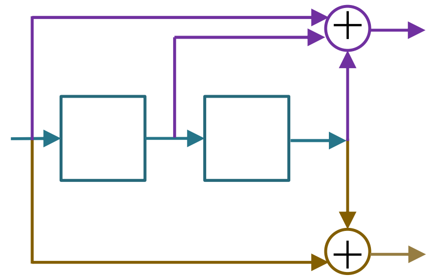


Trellis Diagram

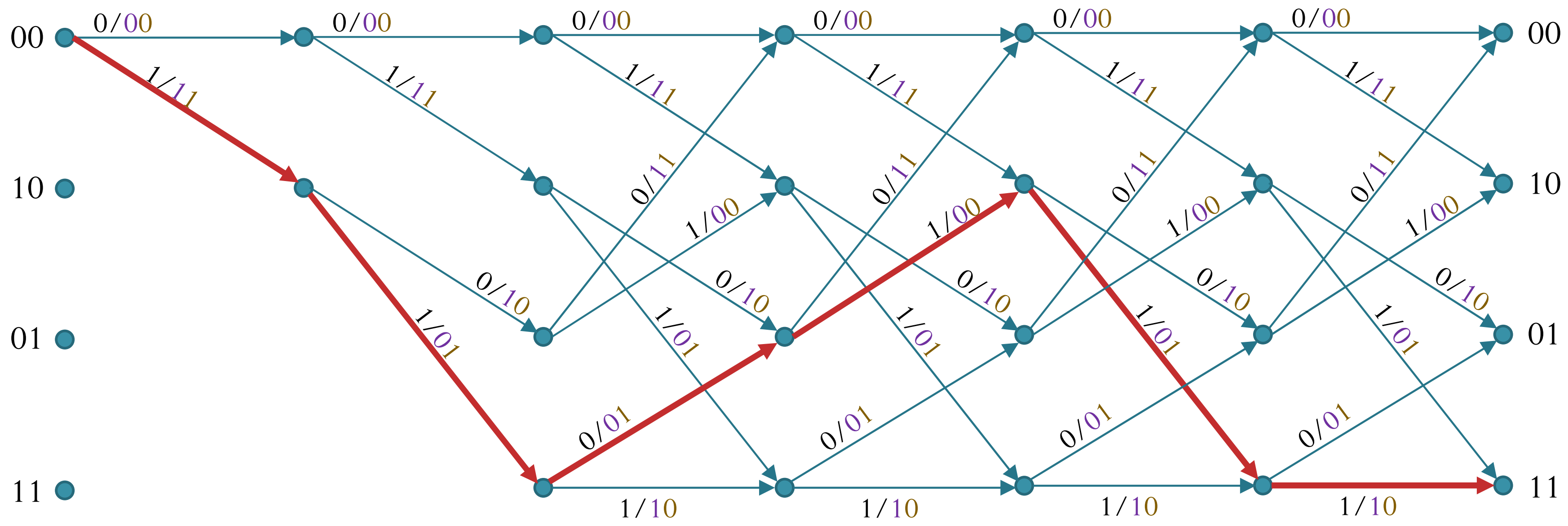


Initially, we always assume that all the contents of the register are 0.

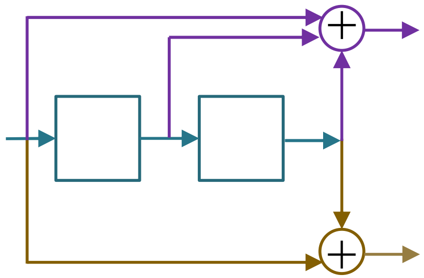
Each path that traverses through the trellis represents a valid codeword.



Trellis Diagram



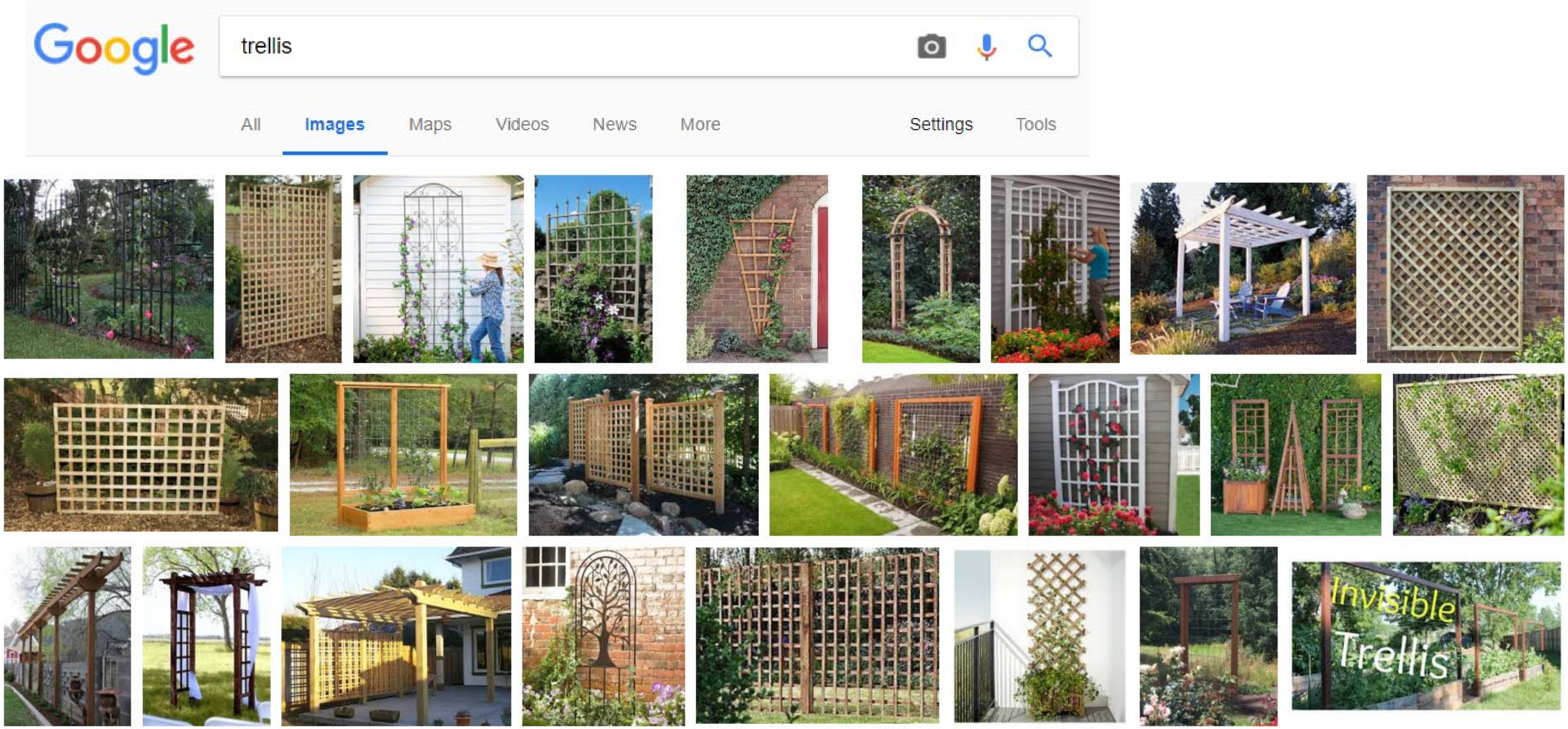
Input	1	1	0	1	1	1
Output	11	01	01	00	01	10



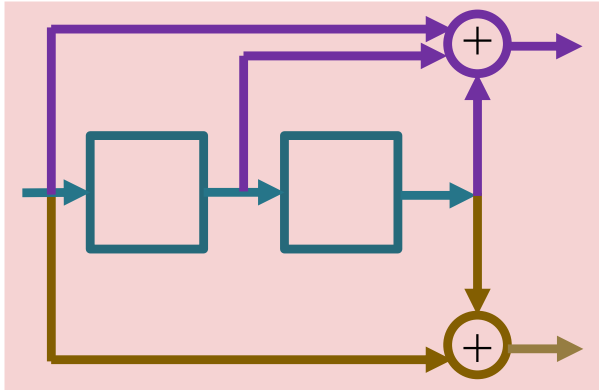
Trellis

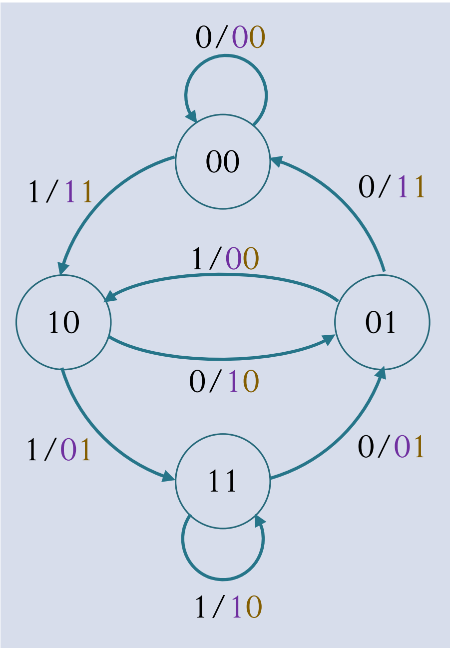
trel·lis /'trɛlɪs/ *n.* [C] a wooden frame for supporting climbing plants

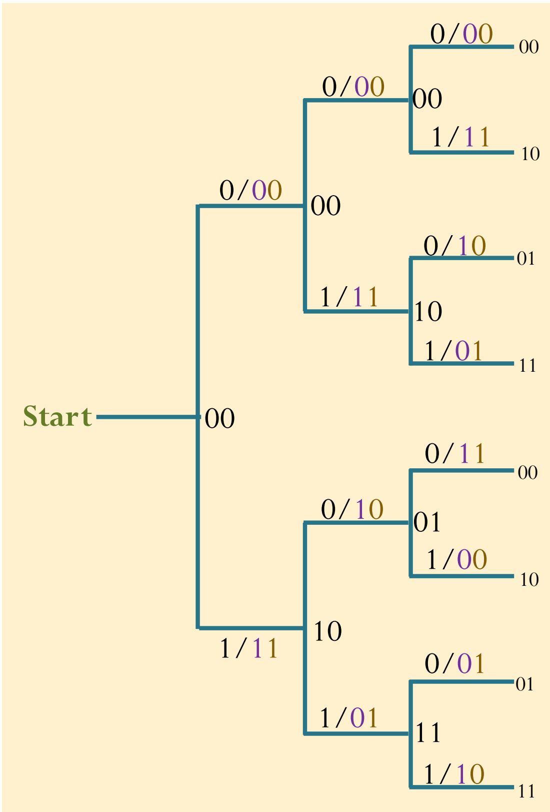
โครงลูกไม้,โครงสร้างบังตาที่เป็นช่อง,โครงสร้างสำหรับปลูกไม้เลื้อย

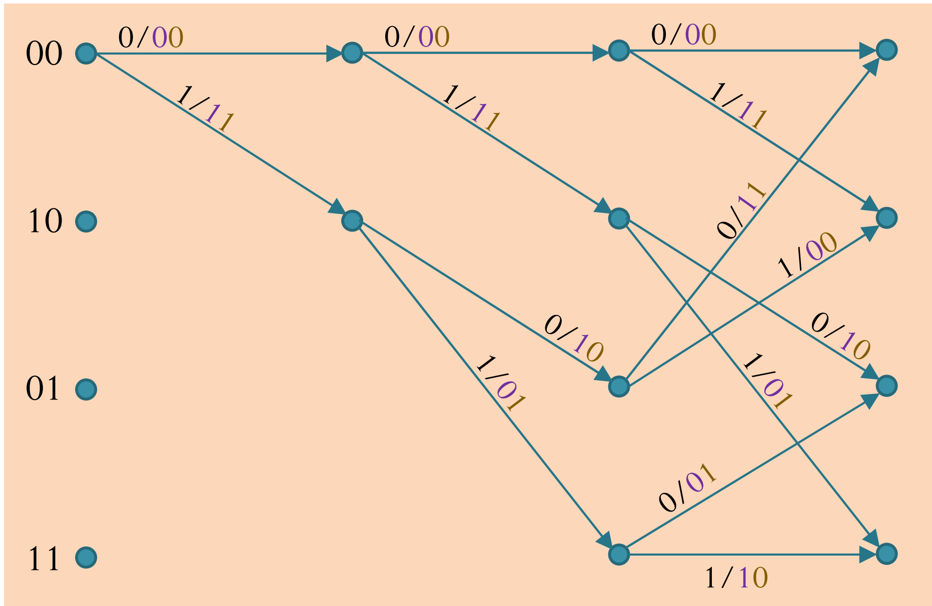


Convolutional Encoder: Graphical Representations

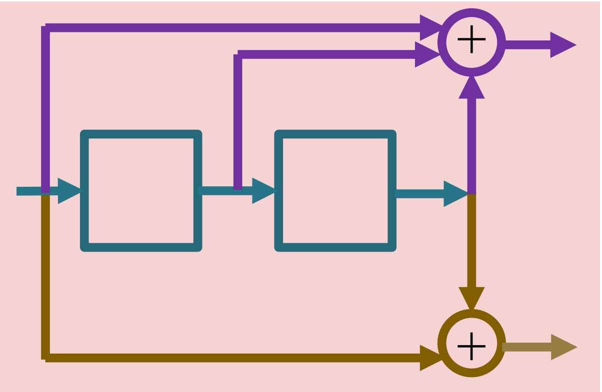


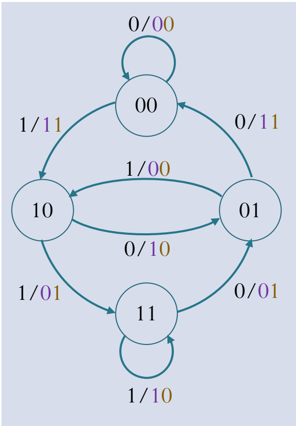


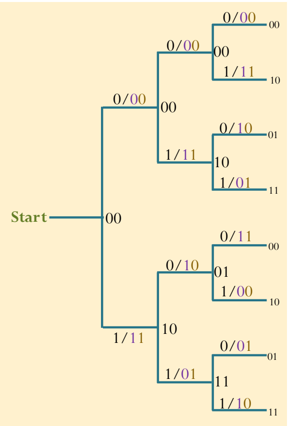


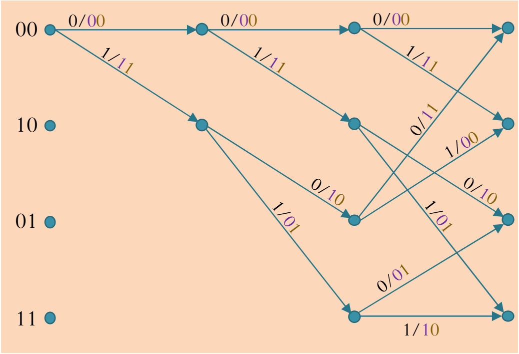


The Codebook









b			**x**					
b_1	b_2	b_3	x_1	x_2	x_3	x_4	x_5	x_6
0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	1	1
0	1	0	0	0	1	1	1	0
0	1	1	0	0	1	1	0	1
1	0	0	1	1	1	0	1	1
1	0	1	1	1	1	0	0	0
1	1	0	1	1	0	1	0	1
1	1	1	1	1	0	1	1	0

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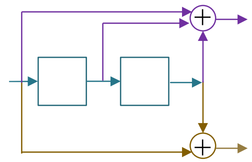
5.2 Binary Convolutional Codes

Introduction to Viterbi Decoding

Convolutional Encoder and Decoder



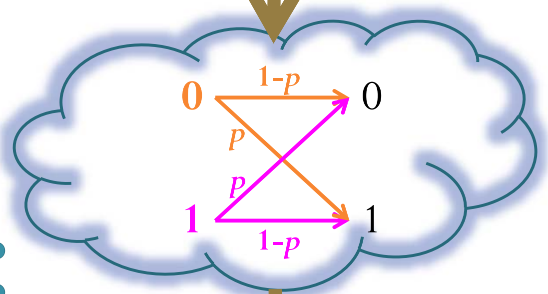
$\underline{\mathbf{d}}, \underline{\mathbf{b}}$

Message (Stream of data)

Convolutional Encoder

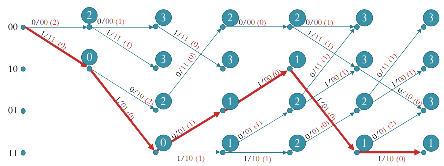
$\underline{\mathbf{x}}, \underline{\mathbf{c}}$

Encoded stream



Binary Symmetric Channel with $p < 0.5$

$$\underline{\mathbf{y}} = \underline{\mathbf{x}} \oplus \underline{\mathbf{e}}$$



$\hat{\underline{\mathbf{d}}}, \hat{\underline{\mathbf{b}}}$

Recovered Message

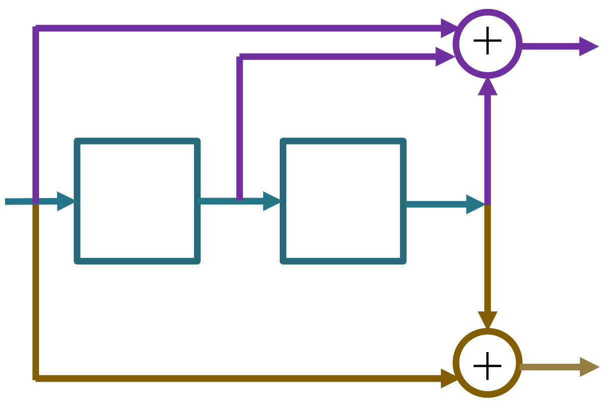
Channel Decoder

minimum distance decoder

Viterbi decoder

Direct Minimum Distance Decoding

- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.
    - $\hat{\underline{\mathbf{x}}} = \arg\min_{\underline{\mathbf{x}}} d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$



Direct Minimum Distance Decoding

$\underline{\mathbf{y}}$					
y_1	y_2	y_3	y_4	y_5	y_6
1	1	0	1	1	1

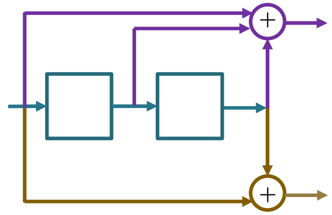
$\underline{\mathbf{b}}$			$\underline{\mathbf{x}}$						
b_1	b_2	b_3	x_1	x_2	x_3	x_4	x_5	x_6	$d(\underline{\mathbf{x}},\underline{\mathbf{y}})$
0	0	0	0	0	0	0	0	0	5
0	0	1	0	0	0	0	1	1	3
0	1	0	0	0	1	1	1	0	4
0	1	1	0	0	1	1	0	1	4
1	0	0	1	1	1	0	1	1	2
1	0	1	1	1	1	0	0	0	4
1	1	0	1	1	0	1	0	1	1
1	1	1	1	1	0	1	1	0	1

$$\hat{\underline{\mathbf{x}}} = \underset{\underline{\mathbf{x}}}{\operatorname{argmin}}\, d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$$

$$= 110101 \text{ or } 110110$$

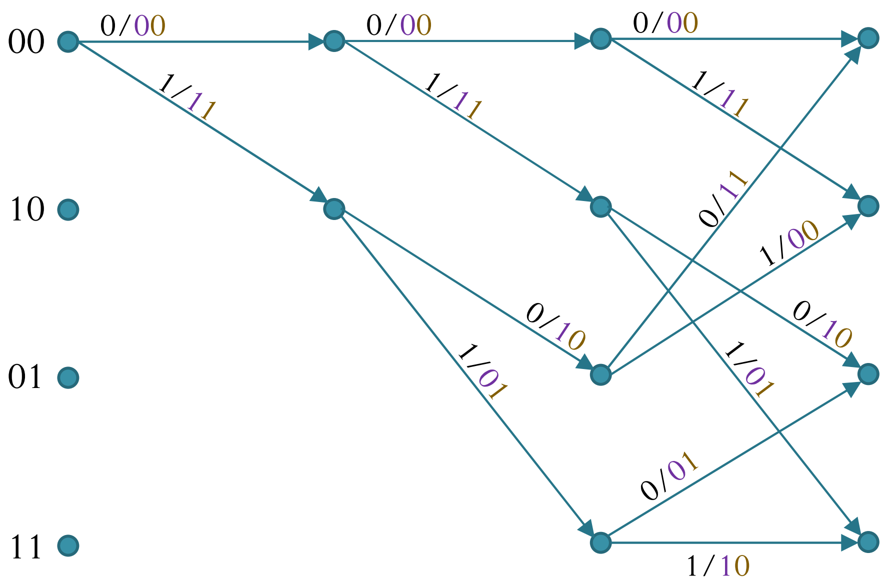
$$\hat{\underline{\mathbf{b}}} = 110 \text{ or } 111$$

Direct Minimum Distance Decoding



- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.

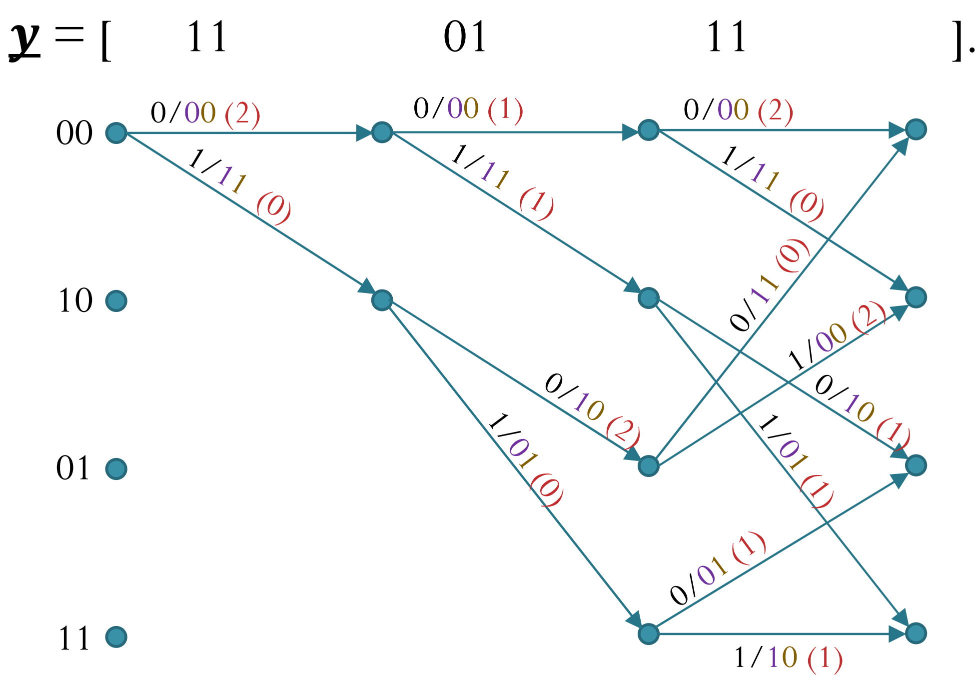
$\underline{\boldsymbol{y}} = [\quad 11 \qquad 01 \qquad 11 \quad].$



For 3-bit message,
there are $2^3 = 8$ possible codewords.
We can list all possible codewords.
However, here, let's first try to work on the distance directly.

Direct Minimum Distance Decoding

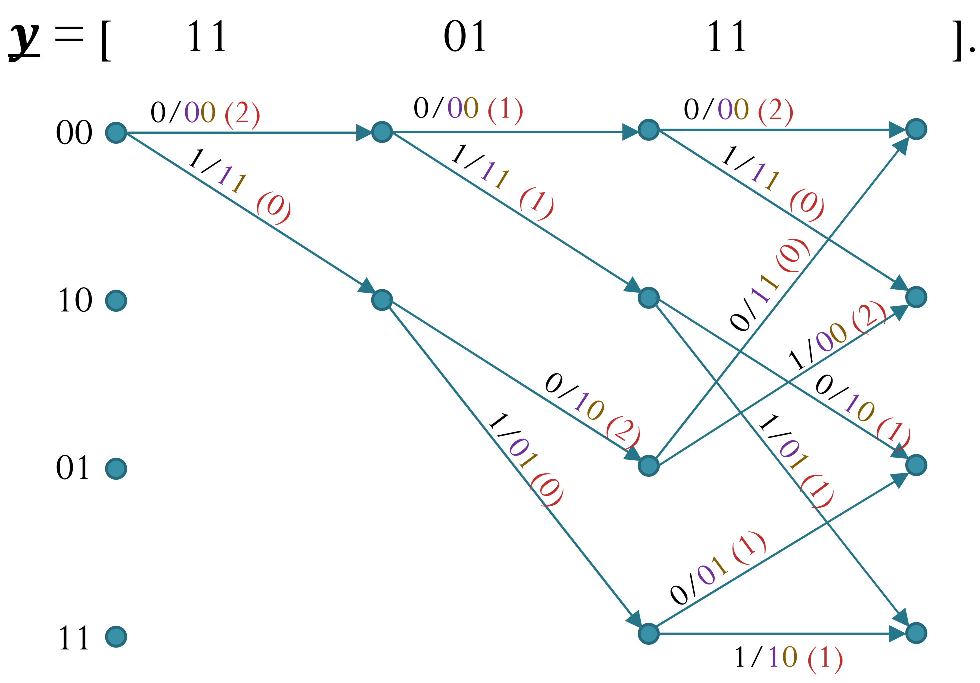
- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



The number in parentheses on each branch is the branch metric, obtained by counting the differences between the encoded bits and the corresponding bits in $\underline{\boldsymbol{y}}$.

Direct Minimum Distance Decoding

- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



$\underline{\mathbf{b}}$	$d(\underline{\mathbf{x}},\underline{\mathbf{y}})$
000	2+1+2 = 5
001	2+1+0 = 3
010	2+1+1 = 4
011	2+1+1 = 4
100	0+2+0 = 2
101	0+2+2 = 4
110	0+0+1 = 1
111	0+0+1 = 1

Viterbi decoding

- Developed by Andrew J. Viterbi
  - Also co-founded Qualcomm Inc.
- Published in the paper "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm", IEEE Transactions on Information Theory, Volume IT-13, pages 260-269, in April, **1967**.

https://en.wikipedia.org/wiki/Andrew_Viterbi

Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm

ANDREW J. VITERBI, SENIOR MEMBER, IEEE

Abstract—The probability of error in decoding an optimal convolutional code transmitted over a memoryless channel is bounded from above and below as a function of the constraint length of the code. For all but pathological channels the bounds are asymptotically (exponentially) tight for rates above R_0, the computational cutoff rate of sequential decoding. As a function of constraint length the performance of optimal convolutional codes is shown to be superior to that of block codes of the same length, the relative improvement increasing with rate. The upper bound is obtained for a specific probabilistic nonsequential decoding algorithm which is shown to be asymptotically optimum for rates above R_0 and whose performance bears certain similarities to that of sequential decoding algorithms.

Manuscript received May 20, 1966; revised November 14, 1966. The research for this work was sponsored by Applied Mathematics Division, Office of Aerospace Research, U. S. Air Force, Grant AFOSR-700-65.

The author is with the Department of Engineering, University of California, Los Angeles, Calif.

I. Summary of Results

SINCE Elias[1] first proposed the use of convolutional (tree) codes for the discrete memoryless channel, it has been conjectured that the performance of this class of codes is potentially superior to that of block codes of the same length. The first quantitative verification of this conjecture was due to Yudkin[2] who obtained

Viterbi and His Decoding Algorithm

[http://viterbi.usc.edu/about/viterbi/viterbi_video.htm]
[https://www.youtube.com/watch?v=A_xHptEqgp0]

Viterbi and His Decoding Algorithm

I came up with some ideas which were kind of based on a tennis tournament, eliminating losers but always having new players coming to the game.

We knew it was an important piece of work, but of course we had no idea of how it was going to revolutionize communications altogether.

This is original copy of the Viterbi algorithm. I read the paper and I really didn't see the significance of it; needless to say I was certainly wrong.

Viterbi and His Decoding Algorithm

Satellite communications, cellular communications, quite a few other things that we now take for granted, it made them really feasible to use on a large scale.

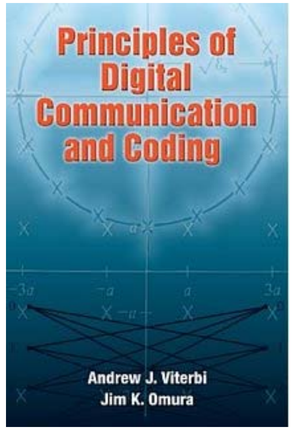
With 45 years of experience and working in communications, [Viterbi algotrithm] is the most important thing I've seen and it's relatively easy to teach.

Viterbi and His Decoding Algorithm

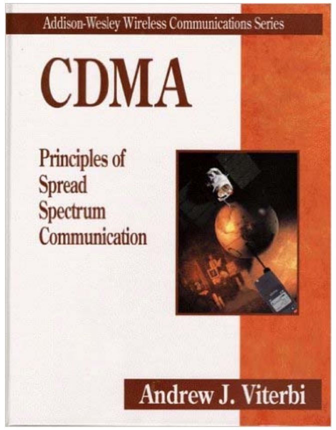
I teach the Viterbi algorithm in several classes … and I can even teach it to undergrads because it's such a simple algorithm in hindsight and it makes such good intuitive sense. It's not just some theoretical concept that never really made it out of the academic community. Every cell phone in use today has at least one Viterbi processor in it and sometimes more.

Other things that you might not associate with communications like the disk drive in your computer has a Viterbi detector in it.

Andrew J. Viterbi



- 1991: Claude E. Shannon Award
- 1952-1957: MIT BS & MS
  - Studied electronics and communications theory under such renowned scholars as Norbert Wiener, Claude Shannon, Bruno Rossi, and Roberto Fano.



- 1962: Earned one of the first doctorates in electrical engineering granted at the University of Southern California (USC)
  - Ph.D. dissertation: error correcting codes
- 2004: USC Viterbi School of Engineering

named in recognition of his $52 million gift

Andrew J. Viterbi

This donation from the Viterbi is the largest naming gift ever received by an American engineering school… Thanks to their gift of 52 million dollars, the USC school of engineering will become the USC Andrew and Erna Viterbi school of engineering.

[http://viterbi.usc.edu/about/viterbi/viterbi_video.htm]
[https://www.youtube.com/watch?v=A_xHptEqgp0]

Andrew J. Viterbi

- Cofounded Qualcomm
- Helped to develop the CDMA standard for cellular networks.
- 1998 Golden Jubilee Award for Technological Innovation
  - To commemorate the 50th Anniversary of Information Theory
  - Given to the authors of discoveries, advances and inventions that have had a profound impact in the technology of information transmission, processing and compression.

1. Norman Abramson: For the invention of the first random-access communication protocol.
2. Elwyn Berlekamp: For the invention of a computationally efficient algebraic decoding algorithm.
3. Claude Berrou, Alain Glavieux and **Punya Thitimajshima (ปัญญา ฐิติมัชฌิมา)**: For the invention of turbo codes.
4. Ingrid Daubechies: For the invention of wavelet-based methods for signal processing.
5. Whitfield Diffie and Martin Hellman: For the invention of public-key cryptography.
6. Peter **Elias**: For the invention of convolutional codes.
7. G. David Forney, Jr: For the invention of concatenated codes and a generalized minimum-distance decoding algorithm.
8. Robert M. Gray: For the invention and development of training mode vector quantization.
9. David **Huffman**: For the invention of the Huffman minimum-length lossless data-compression code.
10. Kees A. Schouhamer Immink: For the invention of constrained codes for commercial recording systems.
11. Abraham Lempel and Jacob Ziv: For the invention of the Lempel-Ziv universal data compression algorithm.
12. Robert W. Lucky: For the invention of pioneering adaptive equalization methods.
13. Dwight O. North: For the invention of the matched filter.
14. Irving S. Reed: For the co-invention of the Reed-Solomon error correction codes.
15. Jorma Rissanen: For the invention of arithmetic coding.
16. Gottfried Ungerboeck: For the invention of trellis coded modulation.
17. Andrew J. **Viterbi**: For the invention of the Viterbi algorithm.

[http://www.itsoc.org/honors/golden-jubilee-awards-for-technological-innovation]

Digital Communication Systems ECS 452

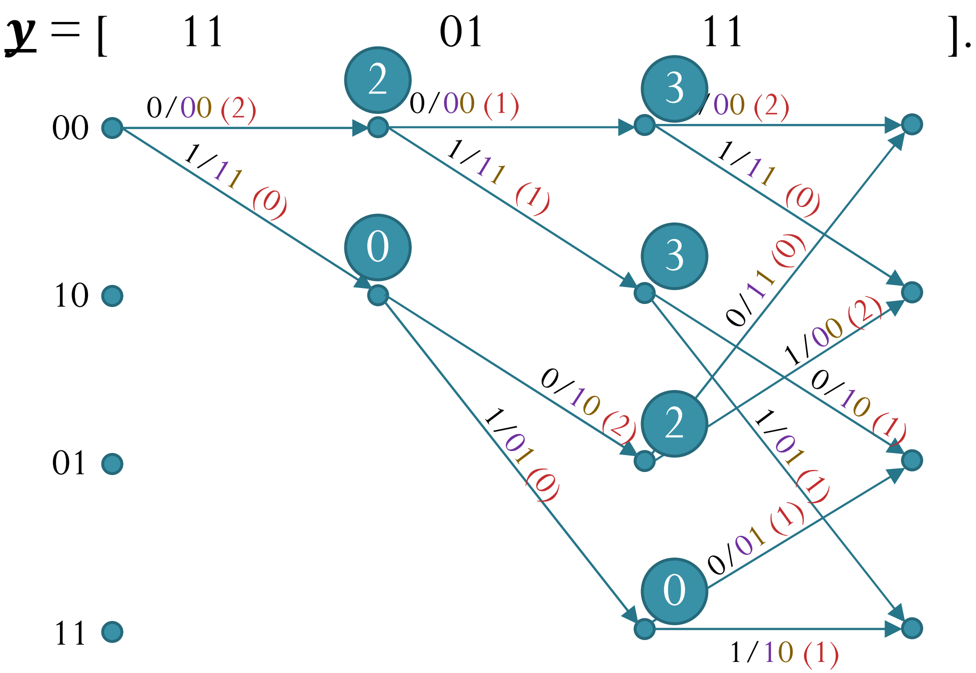
Asst. Prof. Dr. Prapun Suksompong
prapun@siit.tu.ac.th

5.2 Binary Convolutional Codes

Viterbi Decoding

Viterbi Decoding: Ex. 1

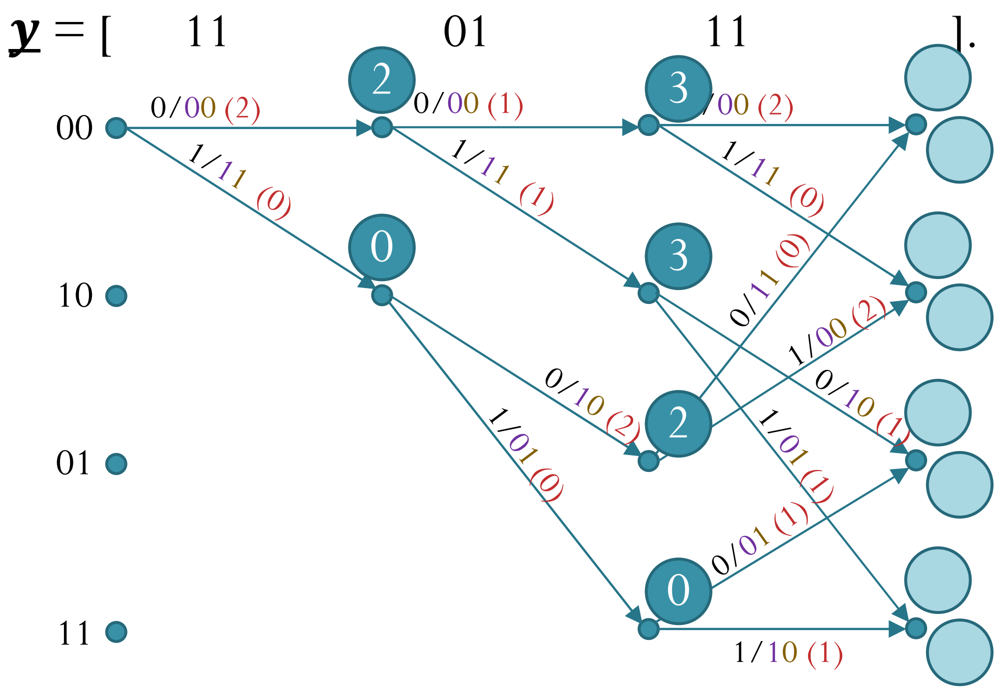
- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



Each **circled number** at a node is the running (cumulative) path metric, obtained by summing branch metrics (distance) up to that node. Here, it is simply the cumulative distance.

Viterbi Decoding: Ex. 1

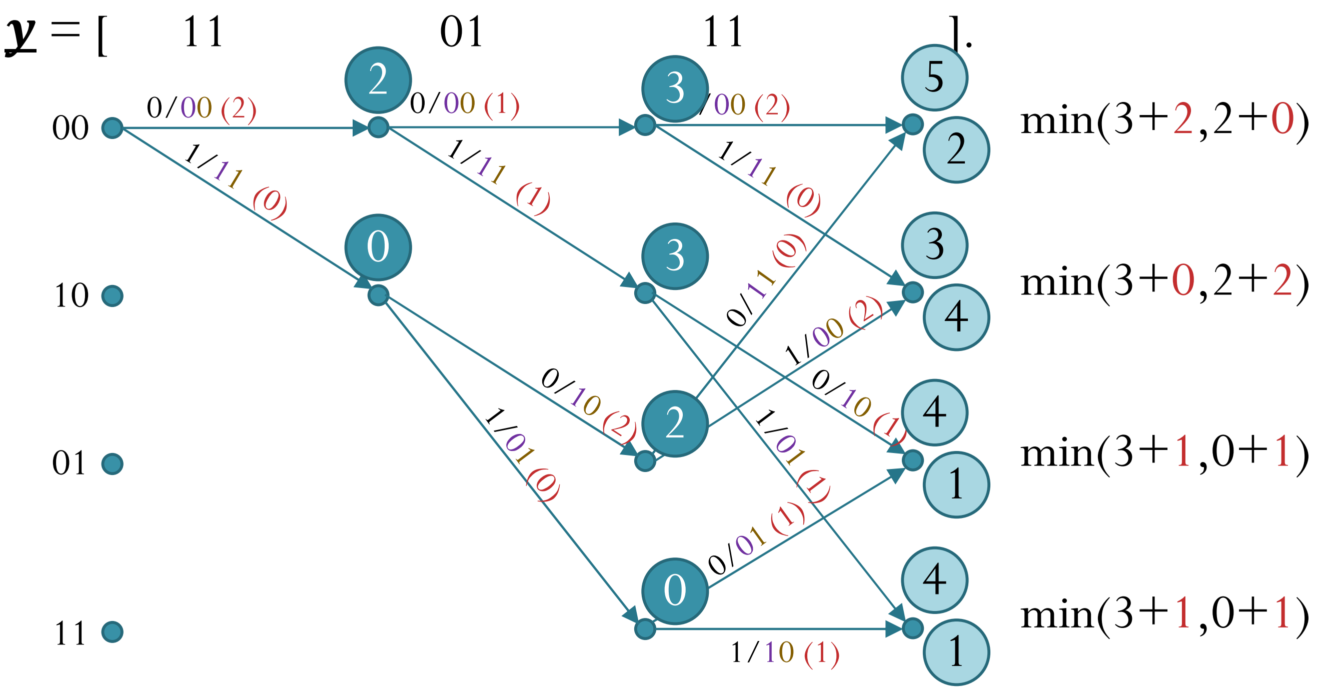
- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



- For the last column of nodes, each of the nodes has two branches going into it.
- So, there are two possible cumulative distance values.

Viterbi Decoding: Ex. 1

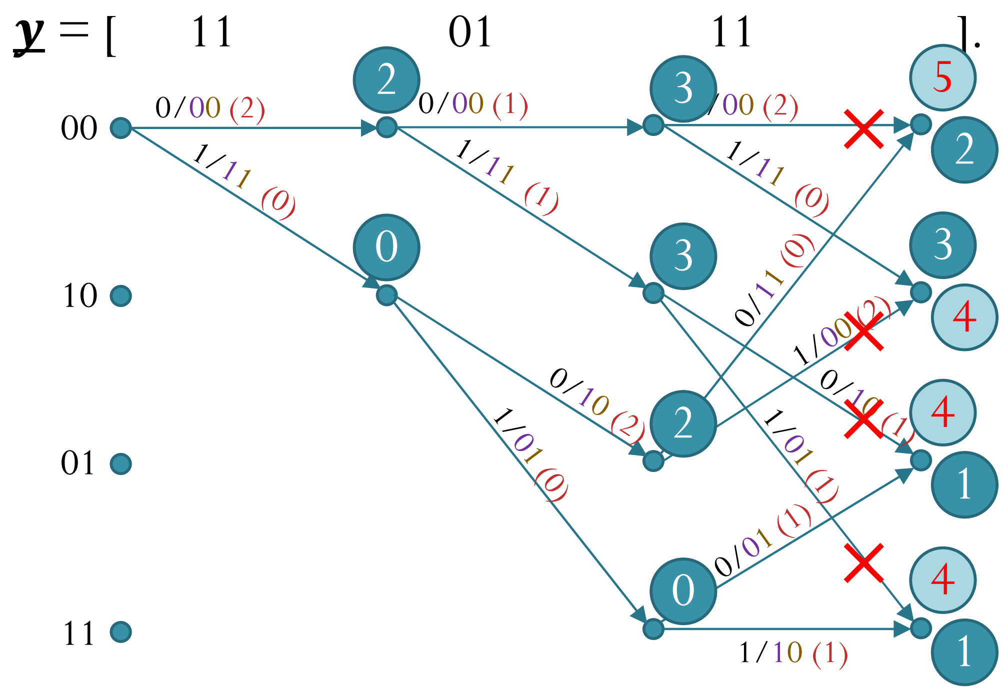
- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



We **discard the larger-distance path** because, regardless of what happens subsequently, this path will have a larger Hamming distance from $\underline{\mathbf{y}}$.

Viterbi Decoding: Ex. 1

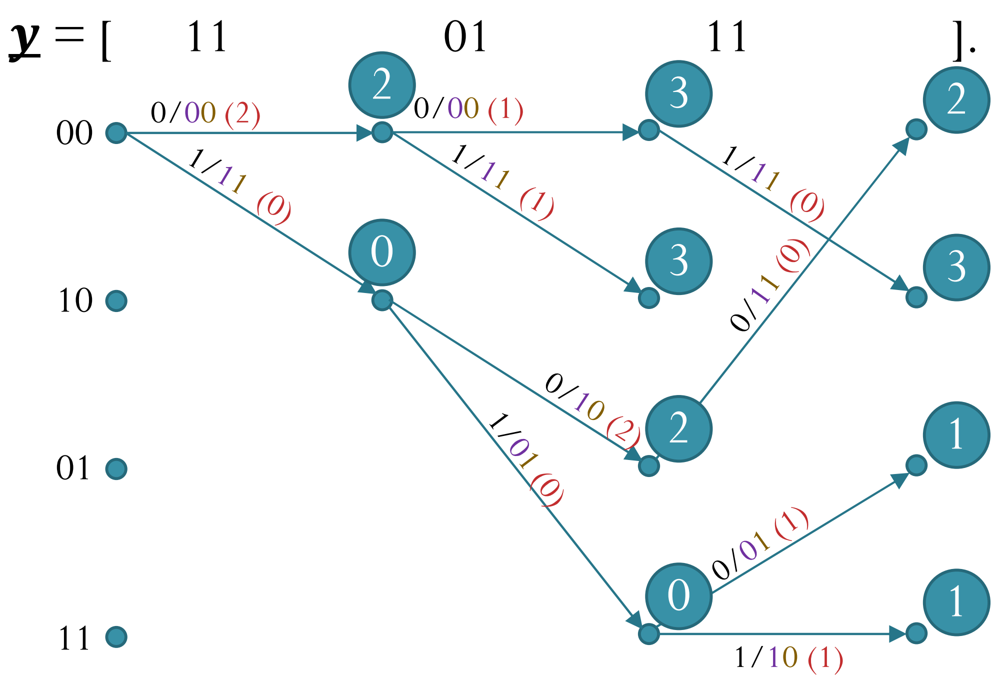
- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



- For the last column of nodes, each of the nodes has two branches going into it.
- So, there are two possible cumulative distance values.
- We **discard the larger-distance path** because, regardless of what happens subsequently, this path will have a larger Hamming distance from $\underline{\mathbf{y}}$.

Viterbi Decoding: Ex. 1

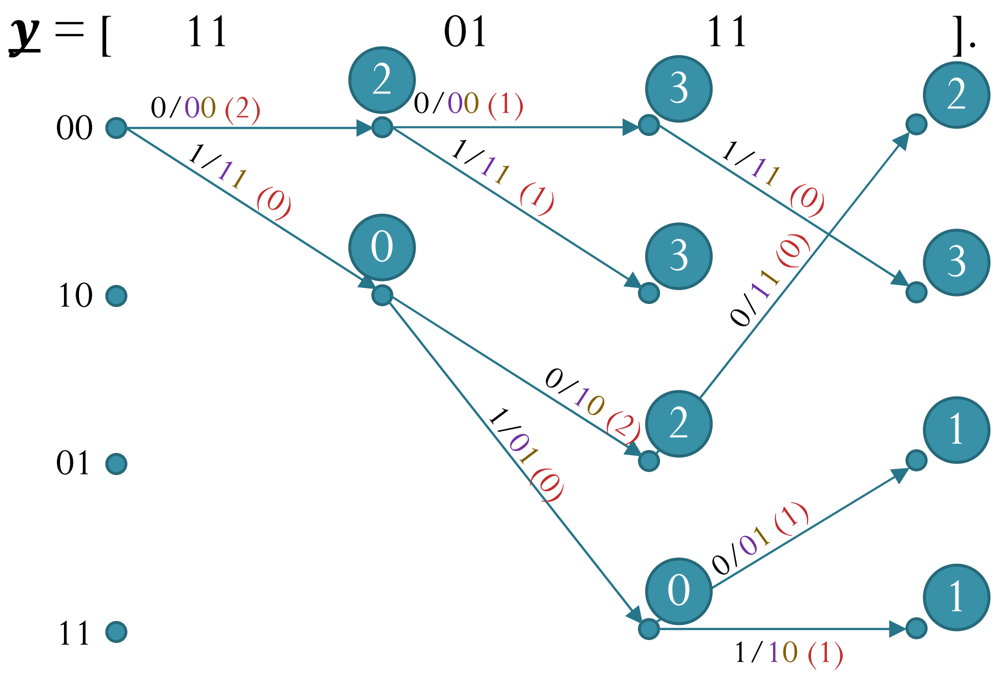
- Suppose $\underline{\mathbf{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\mathbf{y}}$.



- For the last column of nodes, each of the nodes has two branches going into it.
- So, there are two possible cumulative distance values.
- We **discard the larger-distance path** because, regardless of what happens subsequently, this path will have a larger Hamming distance from $\underline{\mathbf{y}}$.

Viterbi Decoding: Ex. 1

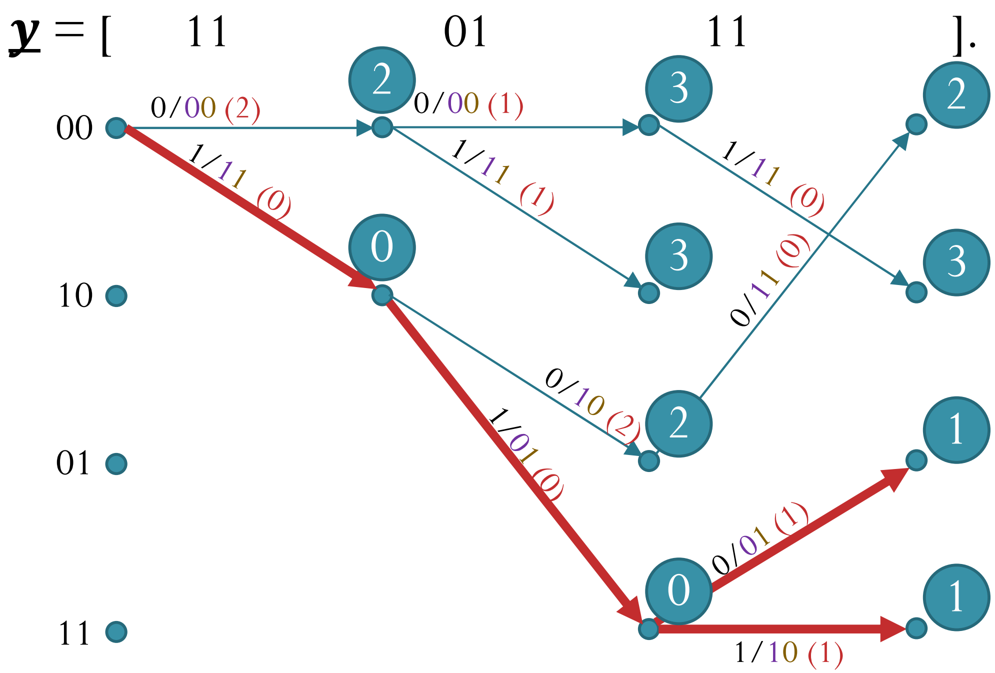
- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



Note that we keep exactly one (optimal) **survivor path** to each state. (Unless there is a tie, then we keep both or choose any.)

Viterbi Decoding: Ex. 1

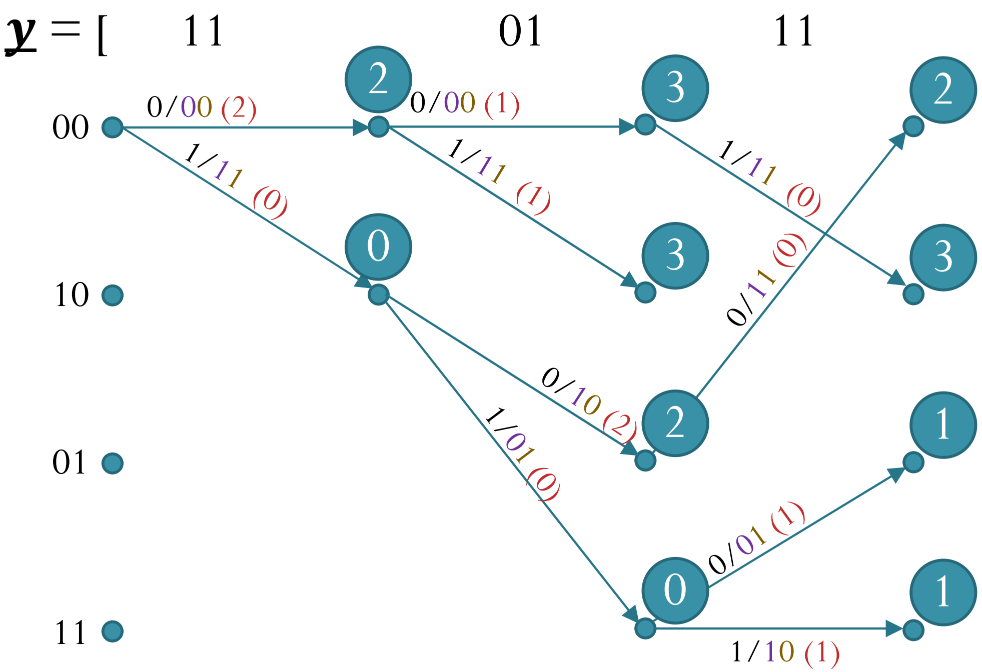
- Suppose $\underline{\boldsymbol{y}}$ = [11 01 11].
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



- So, the codewords which are nearest to $\underline{\boldsymbol{y}}$ is [11 01 01] or [11 01 10].
- The corresponding messages are [110] or [111], respectively.

Viterbi Decoding: Ex. 2

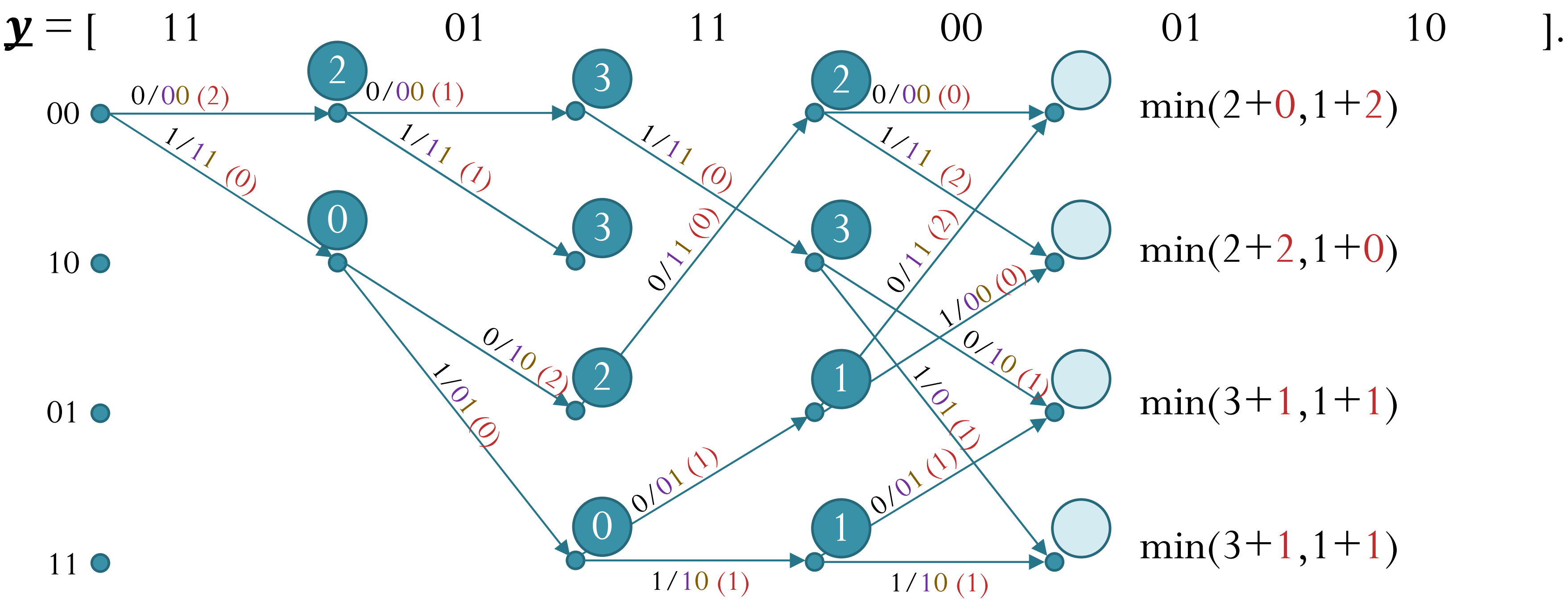
- Suppose $\underline{\boldsymbol{y}} = [\underbrace{11\ 01\ 11}_{\text{same as before}}\ 00\ 01\ 10]$.
- Find $\underline{\hat{\mathbf{b}}}$.



The first part is the same as before. So, we simply copy the diagram that we had.

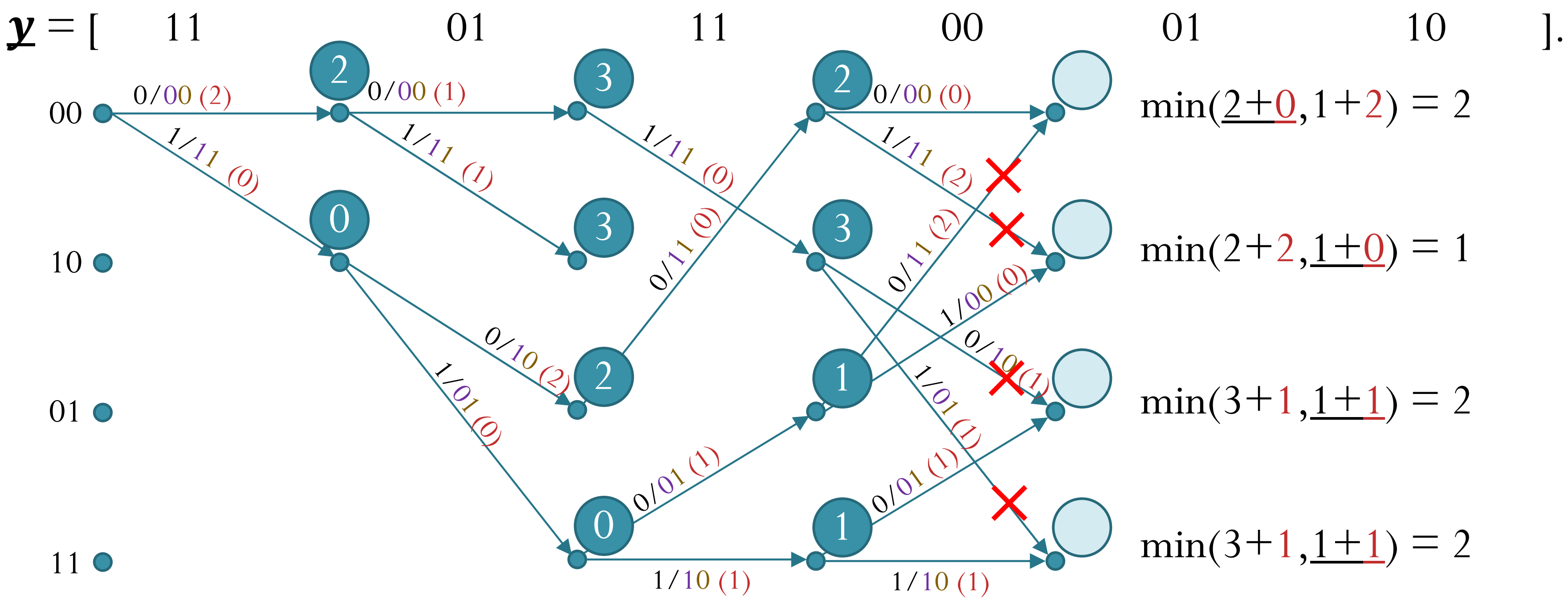
Viterbi Decoding: Ex. 2

- Suppose $\underline{\boldsymbol{y}} = [\underbrace{11\ 01\ 11}_{\text{same as before}}\ 00\ 01\ 10]$.
- Find $\underline{\hat{\mathbf{b}}}$.



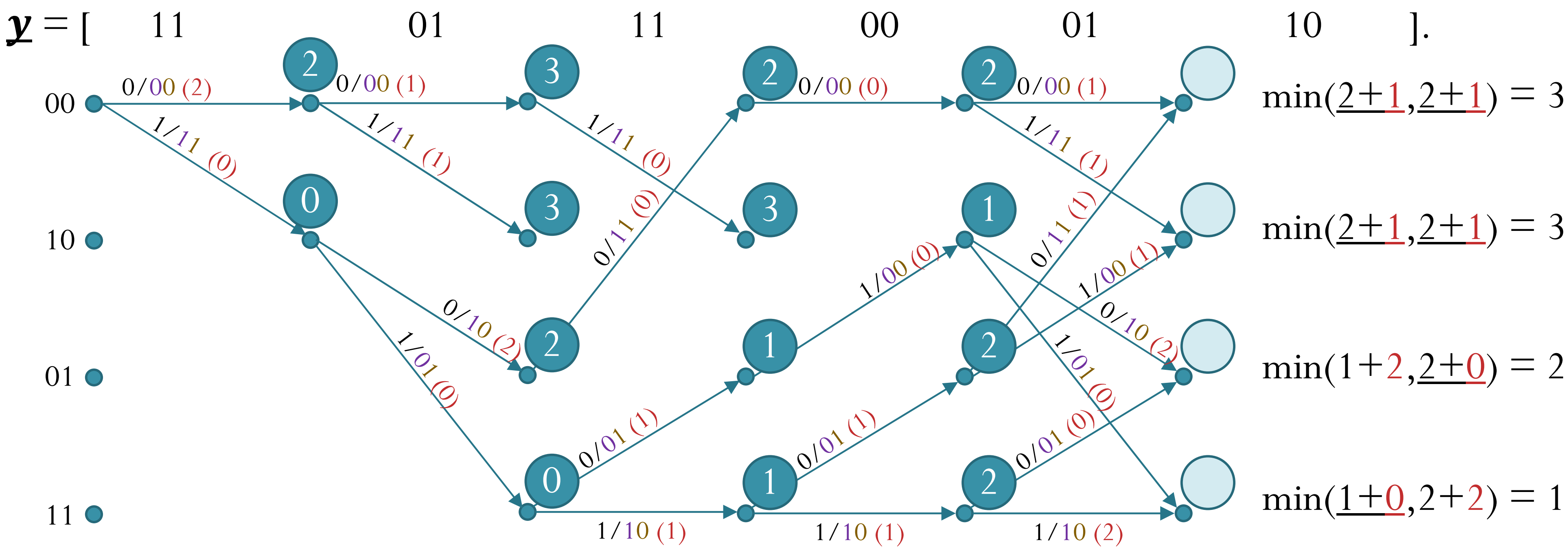
Viterbi Decoding: Ex. 2

- Suppose $\underline{\boldsymbol{y}}$ = [11 01 11 00 01 10].
- Find $\hat{\underline{\mathbf{b}}}$.



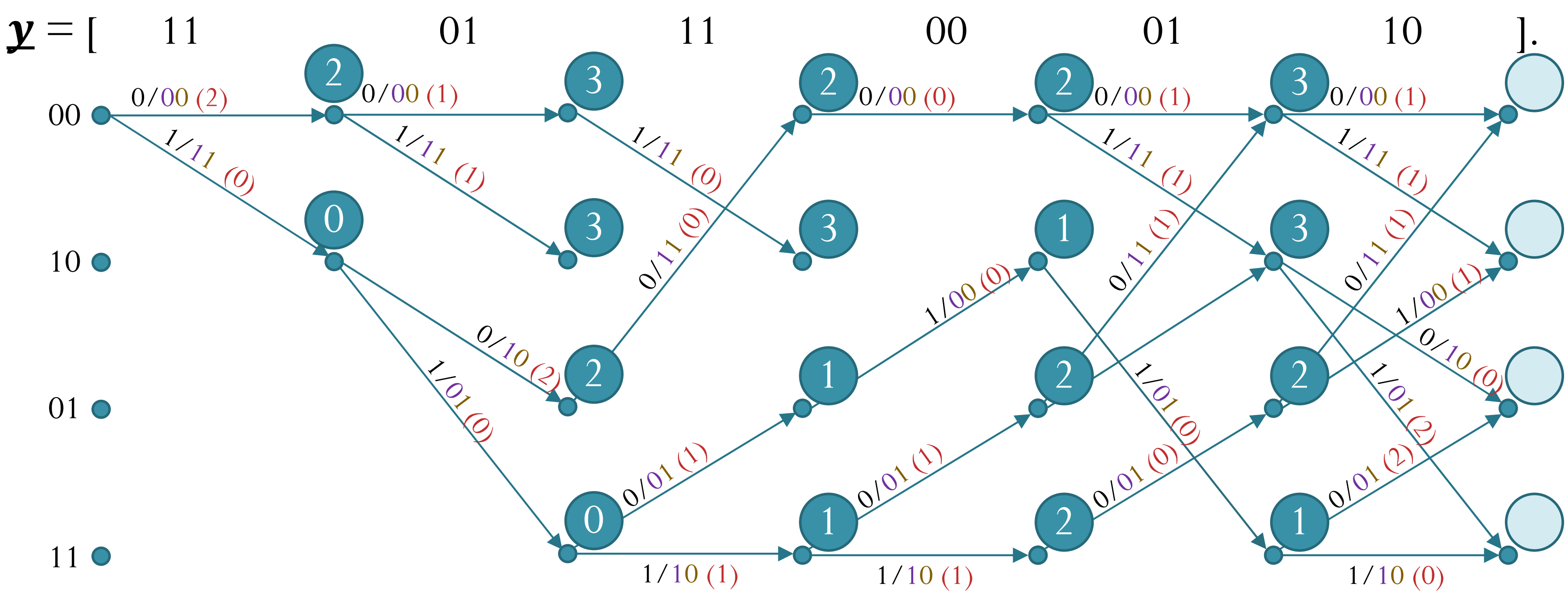
Viterbi Decoding: Ex. 2

- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.



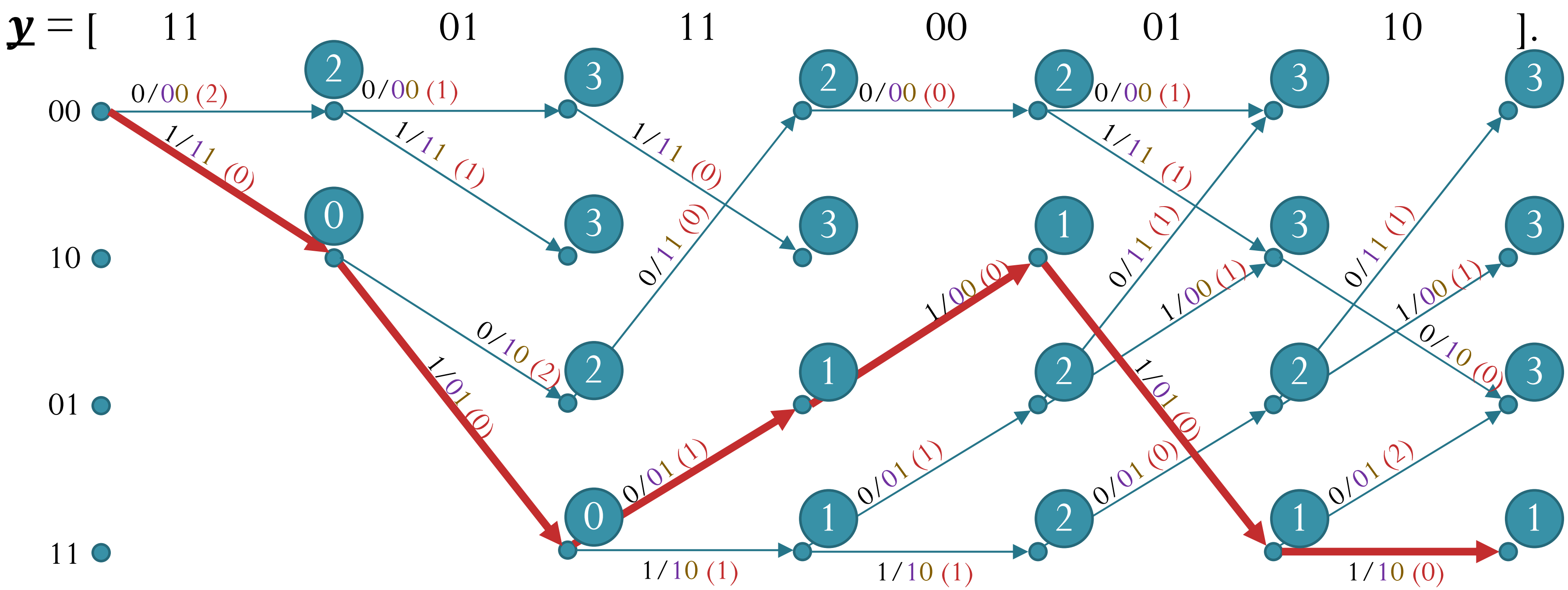
Viterbi Decoding: Ex. 2

- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.



Viterbi Decoding: Ex. 2

- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.

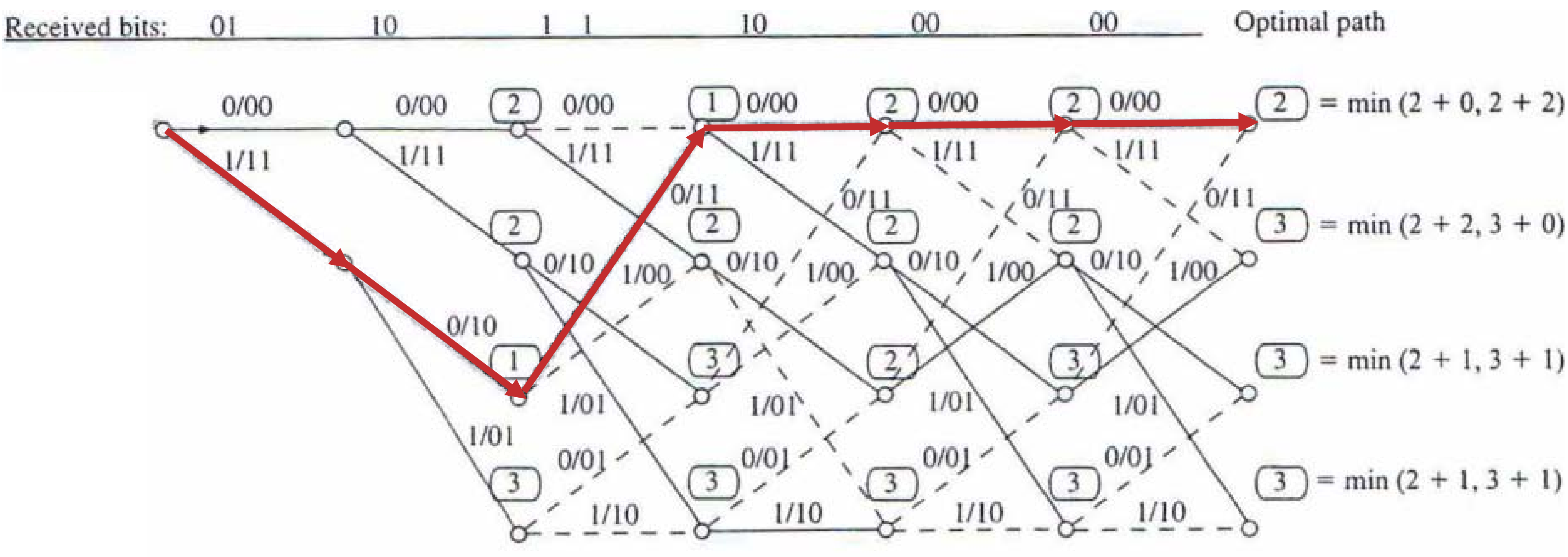


$\hat{\underline{\mathbf{x}}} = [11\ 01\ 01\ 00\ 01\ 10]$

$\hat{\underline{\mathbf{b}}} = [1\ 1\ 0\ 1\ 1\ 1]$

Viterbi Decoding: Ex. 3

- Suppose $\underline{\boldsymbol{y}}$ = [01 10 11 10 00 00].

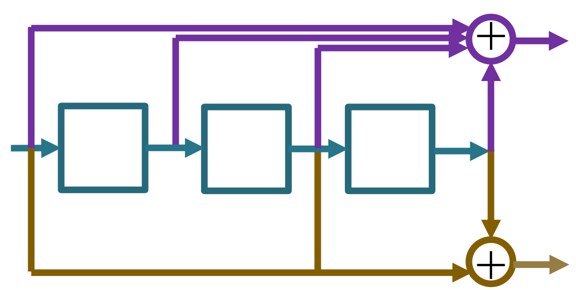


$\hat{\underline{\mathbf{x}}}$ = [11 10 11 00 00 00]

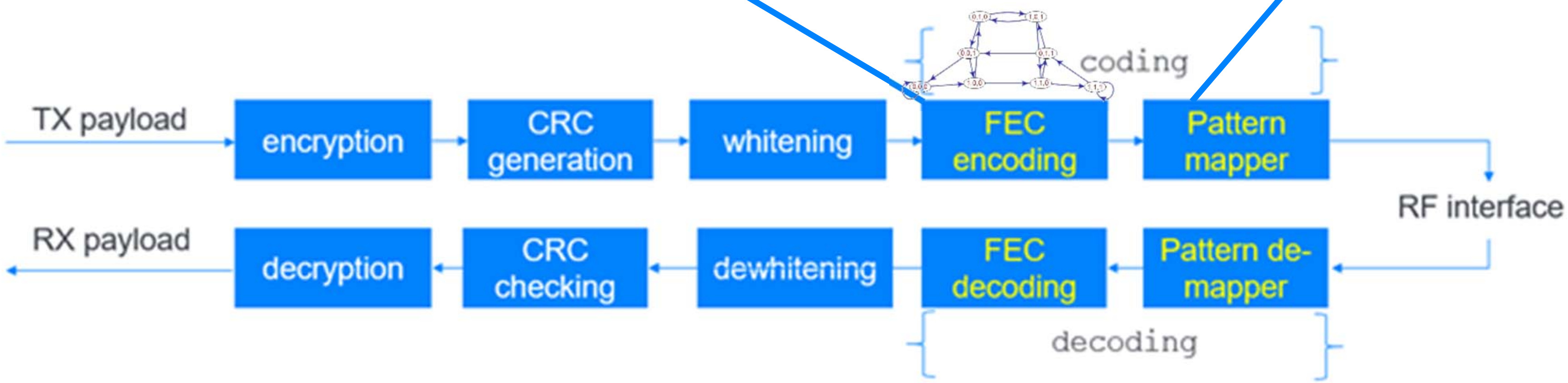
$\hat{\underline{\mathbf{b}}}$ = [1 0 0 0 0 0]

Bluetooth 5

Bluetooth low energy at version 4 does not perform error correction, only error detection. Bluetooth 5 introduces an error correction capability.



Input Bit	Output	
	Option #1	Option #2
0	0	0011
1	1	1100



https://www.bluetooth.com/blog/exploring-bluetooth-5-going-the-distance/
https://www.allaboutcircuits.com/technical-articles/long-distance-bluetooth-low-energy-bit-data-paths/

References: Conv. Codes

- Lathi and Ding, ***Modern Digital and Analog Communication Systems***, 2009
  - [TK5101 L333 2009]
  - Section 15.6 p. 932-941
- Carlson and Crilly, ***Communication Systems: An Introduction to Signals and Noise in Electrical Communication***, 2010
  - [TK5102.5 C3 2010]
  - Section 13.3 p. 617-637